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MOTION OF GAS BUBBLES IN INTENSE TWO-FREQUENCY SOUND FIELDS. (U)

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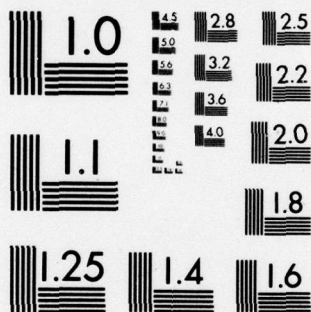


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MOTION OF GAS BUBBLES IN INTENSE  
TWO FREQUENCY SOUND FIELDS

Final Report under Contract N00014-76-C-1000

James R. Church  
Robert A. Hargreaves

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# TABLE OF CONTENTS

	<u>Page</u>
LIST OF FIGURES	v
LIST OF TABLES	
I. INTRODUCTION	1
II. THEORETICAL BACKGROUND	3
III. STUDY PROCEDURE	13
A. The Integration Program	13
B. Data Base	14
IV. RESULTS OF THE NUMERIC INTEGRATIONS	17
A. Small Bubbles ( $f_o \gg f_1, f_2$ )	17
B. Medium Size Bubbles ( $f_o > f_1, f_2$ ; $f_o < f_1 + f_2$ )	25
C. Large Bubbles ( $f_o < f_1, f_2$ )	37
D. Effective Resonance Frequency	37
V. SUMMARY	43
ACKNOWLEDGMENTS	45
APPENDIX - MAGNETIC TAPE FORMATS	47

# LIST OF FIGURES

<u>Figure</u>	<u>Title</u>	<u>Page</u>
1	Bubble Dynamics Differential Equations	4
2	Frequencies Used in Study	9
3	Radial Motion Spectrum of 20 $\mu\text{m}$ Bubble	18
4	Radial Motion Spectrum of 20 $\mu\text{m}$ Bubble	20
5	Radial Motion Spectrum of 20 $\mu\text{m}$ Bubble	22
6	Radial Motion Spectrum of 20 $\mu\text{m}$ Bubble	23
7	Radial Motion Spectrum of 20 $\mu\text{m}$ Bubble	24
8	Radial Motion Spectrum of 20 $\mu\text{m}$ Bubble	26
9	Radial Motion Spectrum of 20 $\mu\text{m}$ Bubble	27
10	Radial Motion Spectrum of 20 $\mu\text{m}$ Bubble	28
11	Radial Motion Spectrum of 20 $\mu\text{m}$ Bubble	29
12	Radial Motion Spectrum of 20 $\mu\text{m}$ Bubble	30
13	Radial Motion Spectrum of 500 $\mu\text{m}$ Bubble	32
14	Radial Motion Spectrum of 500 $\mu\text{m}$ Bubble	33
15	Radial Motion Spectrum of 500 $\mu\text{m}$ Bubble	34
16	Radial Motion Spectrum of 500 $\mu\text{m}$ Bubble	35
17	Radial Motion Spectrum of 500 $\mu\text{m}$ Bubble	38
18	Radial Motion Spectrum of 500 $\mu\text{m}$ Bubble	40
19	Radial Motion Spectrum of 500 $\mu\text{m}$ Bubble	41
20	Radial Motion Spectrum of 500 $\mu\text{m}$ Bubble	42
21	Radial Motion Spectrum of 500 $\mu\text{m}$ Bubble	43
22	Spectrum of Radial Motions of 20 $\mu\text{m}$ Bubble Undergoing FM Slide	45
23	Expanded View of 20 $\mu\text{m}$ Bubble Undergoing FM Slide Oscillations	46



# LIST OF TABLES

<u>Table</u>	<u>Title</u>	<u>Page</u>
I	Summary of Data Base	16
II	Small Bubble Case - Comparison of Integration Results and Perturbation Theory	21
III	Near-Resonance Case - Comparison of Integration Results and Perturbation Theory	36
IV	Large Bubble Case - Comparison of Integration Results and Perturbation Theory	39
AI	GASBUBL5 Identification Record	54



## I. INTRODUCTION

This is the final report, under Contract N00014-76-C-1095, of a study of the response of small air bubbles in water to intense, multifrequency acoustic waves. The approach was to numerically integrate the equation of motion of a bubble using a variety of bubble sizes and acoustic driving pressures. Because the numerical integration required considerable computer time, only a limited number of cases was studied.

Basically, this is a study in bubble dynamics, similar to studies conducted by Flynn<sup>1</sup> and many others.<sup>2</sup> The impetus for the study, however, came from the field of nonlinear acoustics. Westervelt<sup>3</sup> had proposed and Muir and Blue<sup>4</sup> had confirmed that large amplitude sound waves containing two frequencies interact in water to produce secondary sound waves at the sum and difference frequencies. This process is highly inefficient. However, if microscopic bubbles are introduced into the water, the nonlinearity of the medium increases.<sup>5-8</sup> The possibility therefore exists that the efficiency of nonlinear generation of low frequency sound can be improved without seriously degrading all of the process's favorable features.<sup>9</sup> While experimental work<sup>8, 10-13</sup> has verified that this principle operates, most of this work has focused on acoustic measurements and little is known about bubble size distributions and/or bubble densities. The present study was undertaken to partially bridge the gap between theory and experiment.

The method of this study was to numerically integrate the equation of motion of a spherical bubble in a high intensity sound field consisting of two frequencies. Experimental conditions described in Refs. 10 and 12 were modeled. While the motion of a single bubble will not yield numbers which can be directly compared to the experimental data, examination of the motions of bubbles of various sizes can show which sizes are important in the radiation of sound at the difference of the driving frequencies.

This study determines the limits where perturbation theory is valid and indicates quantitatively how results obtained from an exact solution differ from those obtained from perturbation theory at high pressures in complex driving fields.

An outline of the theory of bubble motion, including the perturbation theory solution with two driving frequencies, is given in section II. The numerical integration program and the data base it produced are discussed in section III. The results are presented in section IV, where three cases are examined in detail; a summary is given in section V.

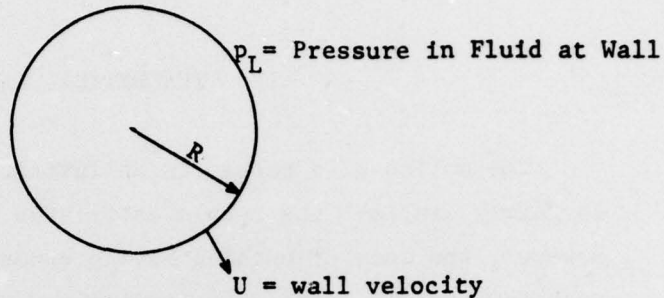
## II. THEORETICAL BACKGROUND

The motion of a bubble in an infinitesimal amplitude sound field is fairly simple: the bubble oscillates at the driving frequency. However, the case of bubbles moving under a large driving pressure is much more complex. Above a driving pressure threshold, the bubbles can go into an almost complete collapse as the momentum of the in-flowing water overcomes the increasing pressure of the compressed gas within the bubble. During these collapses the radius reduces to a very small fraction of its equilibrium value. The precise form this collapse takes depends on the equation of state of the very hot gas within the collapsed bubble and is not well understood.

The situation with a single frequency intense sound field has been studied extensively both analytically<sup>1,14</sup> and numerically.<sup>15,16</sup> The two-frequency case, however, is more difficult to study because of its complexity. Analytical methods, other than in the small amplitude limit where perturbation theory can be applied, do not appear very fruitful. Therefore, "the differential equation" for the bubble motion has been studied numerically. There are, in fact, several differential equations which are used. The four most common ones<sup>1</sup> are shown in Fig. 1:

(1) Noltingk-Neppiras - assumes an incompressible fluid; (2) acoustic approximation - based on allowing for a fixed, finite speed of sound; (3) Herring-Flynn - based on a better accounting of energy storage on compression; and (4) Kirkwood-Bethe - based on specific enthalpy and allowing for the speed of sound to vary with position and time. These various approaches are alternative approximations to the equation of state of the fluid. All give essentially the same result except in the final phase of bubble collapse. To study the broadband high frequency shock waves generated by bubble collapse, the Kirkwood-Bethe equation is required. In addition to an assumption about the equation of state of the fluid, an equation of state for the gas must be assumed. Most investigators





$p_{\infty}$  = Pressure in Fluid  
Far from Wall

$\rho_0$  = Fluid Ambient Density

$c_0$  = Small Signal Speed  
of Sound

$\mu$  = viscosity

INCOMPRESSIBLE FLUID : NOLTINGK-NEPPIRAS

$$R\ddot{U} + \frac{3}{2} U^2 = \frac{1}{\rho_0} [p_L - p_{\infty}]$$

ACOUSTIC APPROXIMATION:

$$R\ddot{U} + \frac{3}{2} U = \frac{1}{\rho_0} [p_L - p_{\infty} + \frac{R}{c_0} (1 - \frac{U}{c_0}) \dot{p}_L]$$

HERRING-FLYNN EQUATION:

$$(1 - 2 \frac{U}{c_0}) R\ddot{U} + (1 - \frac{4}{3} \frac{U}{c_0}) \frac{3}{2} U^2 = \frac{1}{\rho_0} [p_L - p_{\infty} + \frac{R}{c_0} (1 - \frac{U}{c_0}) \dot{p}_L - \frac{4\mu U}{R}]$$

KIRKWOOD-BETHE EQUATION:

$$(1 - \frac{U}{c}) R\ddot{U} + (1 - \frac{U}{3c}) \frac{3}{2} U^2 = (1 - \frac{U}{c}) H + \frac{U}{c} (1 - \frac{U}{c}) R \frac{dH}{dR}$$

$$H \equiv \int_{p_{\infty}}^{p(R)} \frac{dp}{\rho} = \text{specific enthalpy of fluid at bubble wall}$$

$c$  = Local speed of sound

FIGURE 1

BUBBLE DYNAMICS DIFFERENTIAL EQUATIONS



to date have assumed either isothermal or adiabatic conditions for the gas with a specified ratio of specific heats. (Epstein<sup>17</sup> has done some work with adiabatic cases where the ratio of specific heats,  $\gamma$ , takes on a second value during the final stages of collapse.) This study has considered only adiabatic motion.

Whereas this study concentrated on numerical integration of the acoustic approximation equation and not on analytical methods, a perturbation solution for the small amplitude case is useful for comparison purposes. The differential equation considered is

$$R\ddot{R} + \frac{3}{2} \dot{R}^2 = \frac{1}{\rho_o} \left[ p_L - p_\infty + \frac{R}{c_o} \left( 1 - \frac{\dot{R}}{c_o} \right) \dot{p}_L - \frac{4\mu R}{R} \right] , \quad (1)$$

where

$R$  is the bubble radius,

$\rho_o$  is the water density,

$p_L$  is the pressure at the bubble wall,

$p_\infty$  is the ambient pressure,

$c_o$  is the ambient sound speed,

$\mu$  is the viscosity of water,

one dot indicates differentiation with respect to time, and two dots indicate a double differentiation with respect to time. The driving acoustic field,  $p_a$ , will be contained in

$$p_L = p_b + p_a , \quad (2)$$

where

$$p_a = p_1 \sin \omega_1 t + p_2 \sin \omega_2 t , \quad (3)$$

$$p_b = p_\infty \left( \frac{R_o}{R} \right)^{3\gamma} + \frac{2\sigma}{R} . \quad (4)$$

In the above equations,  $p_b$  is the gas pressure in the bubble and consists of the compression term and a term involving the surface tension  $\sigma$ . The value of the adiabatic exponent,  $\gamma$ , could be set arbitrarily in the computer program. In this way adiabatic ( $\gamma=5/3$ ) or isothermal ( $\gamma=1$ ) approximations were made.  $R_0$  is the equilibrium bubble size in the absence of an acoustic sound field and the driving acoustic pressures at angular frequencies  $\omega_1$  and  $\omega_2$  are  $p_1$  and  $p_2$ , respectively.

To obtain the simplest infinitesimal motion approximation, one can examine

$$R\ddot{R} + \frac{3}{2} \dot{R}^2 = \frac{1}{\rho_0} \left\{ p_\infty \left[ 1 - \left( \frac{R_0}{R} \right)^{3\gamma} \right] + p_a \right\} , \quad (5)$$

and let

$$\frac{R}{R_0} = 1 + x , \quad (6)$$

where  $x$  is a small number. If only one driving frequency is used, then the equation in  $x$  to first order would be

$$\ddot{x} + \frac{3\gamma p_\infty}{\rho_0 R_0^2} x = \frac{p_1 \sin \omega_1 t}{\rho_0 R_0^2} . \quad (7)$$

The natural bubble resonance frequency,  $\omega_0$ , in this approximation is given by

$$\omega_0^2 = \frac{3\gamma p_\infty}{\rho_0 R_0^2} , \quad (8)$$

and

$$x = x_{01} \sin \omega_1 t , \quad (9)$$

with

$$x_{01} = \frac{p_1}{\rho_o R_o^2 (\omega_o^2 - \omega_1^2)} = \frac{1}{3\gamma} \left( \frac{\omega_o^2}{\omega_o^2 - \omega_1^2} \right) \frac{p_1}{p_\infty} \quad (10)$$

For two driving frequencies, the perturbation theory solution to first order is

$$x = x_{01} \sin \omega_1 t + x_{02} \sin \omega_2 t \quad , \quad (11)$$

with  $x_{01}$  and  $x_{02}$  given by Eq. (10) with appropriate subscripts. This solution may then be used in an approximation to the equation of motion which contains terms of second order.

At this point one must decide on the relative size of  $x_{01,02}$  and  $R_o/\lambda_{1,2}$  where  $\lambda_{1,2}$  are the wavelengths at  $\omega_{1,2}$ . If the bubble is so small that  $x_{01}$  is larger than this ratio, the terms containing  $R_o \dot{p}_a/c_o$  and  $\dot{x}R_o/c_o$  may be ignored; otherwise they must be retained. For simplicity these terms will be dropped since we are looking only for an illustrative solution.

Letting

$$x = x_F + x_S \quad , \quad (12)$$

$x_F$  being the first order solution and  $x_S$  the second order term, the equation of motion becomes

$$(1+x) \ddot{x} + \omega_o^2 x = \left\{ \frac{1}{\rho_o R_o^2} (p_1 \sin \omega_1 t + p_2 \sin \omega_2 t) \right\} - \frac{3}{2} \dot{x}^2 - \omega_o^2 \frac{(3\gamma+1)}{2} x^2 \quad . \quad (13)$$

The term in braces on the right-hand side of Eq. (13) is balanced by  $x_F$  from the left-hand side, leaving

$$\ddot{x}_S + \omega_o^2 x_S = -x_F \ddot{x}_F - \frac{3}{2} \dot{x}_F^2 + \omega_o^2 \left( \frac{3\gamma+1}{2} \right) x_F^2 \quad , \quad (14)$$



or

$$\begin{aligned}
\ddot{x}_s + \omega_o^2 x_s = & x_{o1}^2 \left\{ \left[ \frac{\omega_1^2}{2} + \left( \frac{3\gamma+1}{4} \right) \omega_o^2 \right] [1 + \cos(2\omega_1 t)] \right. \\
& \left. - \frac{3}{4} \omega_1^2 [1 - \cos(2\omega_1 t)] \right\} \\
& + x_{o2}^2 \left\{ \left[ \frac{\omega_2^2}{2} + \left( \frac{3\gamma+1}{4} \right) \omega_o^2 \right] [1 + \cos(2\omega_2 t)] \right. \\
& \left. - \frac{3}{4} \omega_2^2 [1 - \cos(2\omega_2 t)] \right\} \\
& + x_{o1} x_{o2} \left\{ \left[ \frac{\omega_1^2 + \omega_2^2}{4} - \left( \frac{3\gamma+1}{2} \right) \omega_o^2 \right] [\cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t] \right. \\
& \left. - \frac{3}{2} \omega_1 \omega_2 [\cos(\omega_1 - \omega_2)t - \cos(\omega_1 + \omega_2)t] \right\} .
\end{aligned} \tag{15}$$

This expression contains all of the essential features of a more complete development for infinitesimal waves. A simple harmonic oscillation is driven by terms at 0 Hz,  $2\omega_1$ ,  $2\omega_2$ ,  $\omega_1 + \omega_2$ , and  $\omega_1 - \omega_2$ . This perturbation theory development is incorrectly handling the 0 Hz term. The other forcing terms should be meaningful at low driving pressures when all frequencies are far from resonances. The terms arising from the  $\ddot{R}^2$  and  $R\ddot{R}$  in Eq. (1) are included here but were absent from the analytical development by Rolfeigh and Clyne.<sup>10</sup> In that development the bubbles were assumed very small and consequently  $\omega_o$  was assumed much larger than all other frequencies in the problem. In the current study these assumptions were not made and therefore these terms have been kept.

A diagram of the frequencies studied is shown in Fig. 2. In this diagram,  $f_o$  is the bubble resonance frequency,



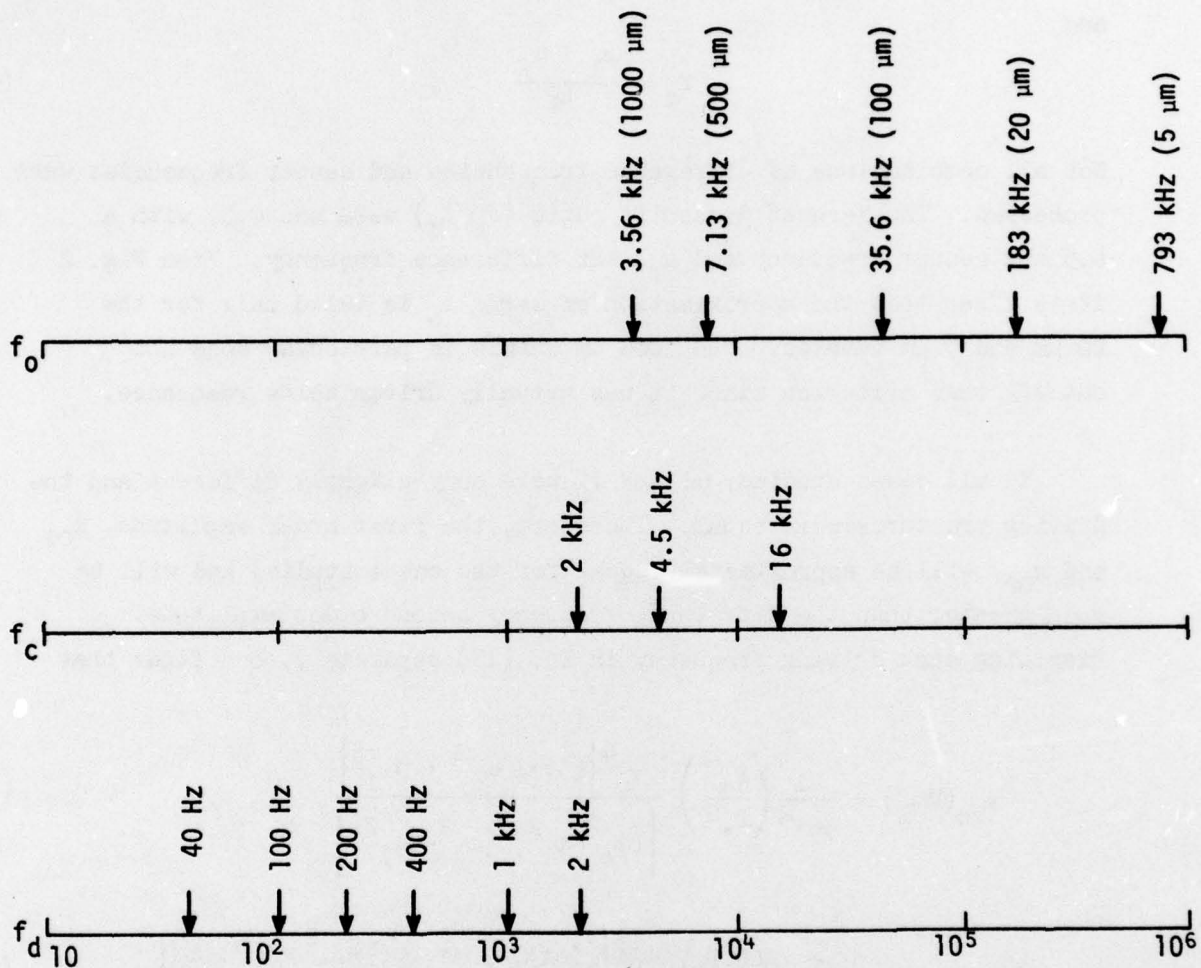


FIGURE 2

FREQUENCIES USED IN STUDY

Difference Frequency  $f_d$

Center Frequency  $f_c$

Bubble Resonance Frequency  $f_0$  (Bubble radius in brackets)

$$f_d = \frac{\omega_1 - \omega_2}{2\pi} , \quad (16)$$

and

$$f_c = \frac{\omega_1 + \omega_2}{4\pi} . \quad (17)$$

Not all combinations of difference frequencies and center frequencies were processed. The largest downshift ratio ( $f_c/f_d$ ) used was 4.5, with a 4.5 kHz center frequency and a 1 kHz difference frequency. From Fig. 2 it is clear that the approximation of large  $\omega_0$  is valid only for the 20  $\mu\text{m}$  and 5  $\mu\text{m}$  bubbles. The 1000  $\mu\text{m}$  bubble in particular does not satisfy this criterion since it was actually driven below resonance.

In all cases studied,  $\omega_1$  and  $\omega_2$  were only slightly different and the driving pressures were equal. Therefore, the first order amplitude,  $x_{01}$  and  $x_{02}$ , will be approximately equal for the cases studied and will be much greater than the difference frequency second order amplitude. Examining each driving frequency in Eq. (15) separately, one finds that

$$x_{so}(2\omega_1) = \frac{1}{36\gamma^2} \left( \frac{p_1^2}{p_\infty^2} \right) \frac{\omega_0^4 [(3\gamma+1)\omega_0^2 + 5\omega_1^2]}{[(\omega_0^2 - \omega_1^2)(\omega_0^2 - 4\omega_1^2)]} , \quad (18)$$

$$x_{so}(\omega_1 + \omega_2) = \frac{1}{18\gamma^2} \left( \frac{p_1 p_2}{p_\infty^2} \right) \frac{\omega_0^4 [(3\gamma+1)\omega_0^2 + (\omega_1^2 + \omega_2^2) + 3\omega_1\omega_2]}{[(\omega_0^2 - \omega_1^2)(\omega_0^2 - \omega_2^2)(\omega_0^2 - (\omega_1 + \omega_2)^2)]} , \quad (19)$$

$$x_{so}(\omega_1 - \omega_2) = \frac{1}{18\gamma^2} \left( \frac{p_1 p_2}{p_\infty^2} \right) \frac{\omega_0^4 [(3\gamma+1)\omega_0^2 + (\omega_1^2 + \omega_2^2) - 3\omega_1\omega_2]}{[(\omega_0^2 - \omega_1^2)(\omega_0^2 - \omega_2^2)(\omega_0^2 - (\omega_1 - \omega_2)^2)]} , \quad (20)$$

and  $x_{so}(2\omega_2)$  can be obtained from Eq. (18) by substituting  $\omega_2$  for  $\omega_1$ . The equation for  $\omega_0$  has been used to put these equations in the form of dimensionless ratios. While these equations can give qualitative results away from resonances and low driving pressures, they are really not applicable at the acoustic pressures usually present in experiments.

For an  $\omega_0$  that is much larger than any of other frequencies involved, these equations reduce to the results of Rolfeigh and Clynch.<sup>10</sup> In that limit the amplitudes of the bubble radial oscillations at the sum and difference frequencies are equal and are twice the value at each second harmonic. Thus, in the large  $\omega_0$  limit, the amplitude oscillations at the secondary frequencies are partitioned into three equal parts: at the sum frequency, at the difference frequency, and at both the second harmonics. However, this partition will not apply to the radiated sound field. The sound radiated will be proportional to the second time derivative of the volume oscillation, causing each frequency component of the sound to be weighted by the frequency squared.

In the general case, the equations are quite complex. Even though they apply only at small acoustic pressures, they can serve as a guide to frequency dependencies. In addition to examining absolute levels, the ratios of  $x_{so}$ 's to the values computed by numerical integration will be compared to provide an estimate of when the perturbation theory solution can be used to predict spectral shape.



### III. STUDY PROCEDURE

The motion of a gas bubble in a two-frequency intense sound field was investigated through numerical integration of an equation of motion. The equation integrated by the computer was the acoustic approximation with viscosity added:

$$R\ddot{R} + \frac{3}{2} \dot{R}^2 = \frac{1}{\rho_0} \left[ P_L - P_\infty + \frac{R}{c_0} \left( 1 - \frac{\dot{R}}{c_0} \right) \dot{P}_L - \frac{4\mu\dot{R}}{R} \right] \quad (21)$$

The output, which was unevenly spaced in time, was first placed on magnetic tape and then was interpolated to produce an even time grid which was Fourier transformed. The output of the fast Fourier transforms (FFT's) was placed on magnetic tapes and plotted. (The capability to plot time histories of bubbles also exists.)

#### A. The Integration Program

Equation (21) was integrated by program GASBUBL5 on the CDC 3200 computer at Applied Research Laboratories, The University of Texas at Austin (ARL:UT). This program accepts control cards which define the characteristics of the bubbles, the ambient fluid, the driving sound field, the initial conditions, and the time interval to integrate. The equation of motion was then integrated using a fourth order Runge-Kutta algorithm. This integration procedure was chosen because it allows the step size to vary. The output from GASBUBL5 is a magnetic tape containing the times, radii, and the first and second time derivatives of R.

The integration time step was varied automatically in the program. An absolute maximum time step of one hundredth of the smallest period (bubble resonance period or driving sound period) was used. A convergence criterion on the change in bubble radius was applied after each step. If this test



is failed, the program backed up one step and halved the step. If the convergence criterion is failed twice consecutively, a major restart was done. In this case the program backed up 20 steps, reduced the time step by a factor of 10, and continued. A circular buffer of the 100 previous points was kept to allow for five overlapping major reductions. Only after a point was 100 calculations old was it moved from the circular buffer to storage for output to magnetic tape. Major reductions in step size were also triggered by a negative bubble radius.

A problem was encountered with this algorithm in the case of 5  $\mu\text{m}$  bubbles undergoing collapse. The step size was constantly being reduced to the point where accumulated round-off error made the calculation meaningless. Therefore, when the step size became less than  $10^{-90}$  sec, the step size was reset at  $10^{-90}$  sec and kept there.

In addition to being reduced automatically, the step size was automatically doubled if 20 steps of the same size occurred. If the resulting step size was greater than the absolute maximum, the maximum was used.

To validate the program, the curves given by Flynn<sup>1,15</sup> were recalculated. This was done for both adiabatic and isothermal conditions. In both cases the GASBUBL5 curves were in excellent agreement with those of Flynn.

#### B. Data Base

Center driving frequencies and difference frequencies which matched experimental data on hand were chosen for study. A wide range of bubble sizes (5  $\mu\text{m}$  to 1000  $\mu\text{m}$ ) was used to include cases where the resonance was above, nearly equal to, and below the same driving frequency. Pressures of 0.1 bar, where the perturbation theory should hold, to 1.5 bars, where the perturbation theory certainly will not hold, were used.

Table I is a summary of the cases that were integrated. In each case marked with an X, the full set of pressures was run. Each integration with two-frequency data was run out to a time corresponding to 100 difference frequency periods. All integrations began with the bubble at rest. In all cases an adiabatic exponent of  $5/3$  was used.

TABLE I  
SUMMARY OF DATA BASE

$f_c$ (kHz)	2					4.5					16				
$f_d$ (Hz)	0	40	100	200	400	0	100	200	400	1000	0	200	400	1000	2000
Bubble Size ( $\mu m$ )															
1000	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
500	X		X		X	X			X		X		X		X
100	P		P		P	X	X		X		X		X		X
20	X		X		X	X	X		X		X		X		X
5	X				X	X			X		X		X		X

Items marked with X have pressures of 0.1, 0.25, 0.50, 1.0, and 1.5 bars complete.

Items marked with P are partially complete.



#### IV. RESULTS OF THE NUMERIC INTEGRATIONS

A thorough analysis of all the integration results has not yet been made. Three illustrative cases will be examined which shed light on the motion of bubbles in a complex high pressure sound field. In all cases an adiabatic equation of state ( $\gamma=5/3$ ) for an ideal gas was used. The ambient pressure was taken at 1.0 bar, the surface tension was 75 dynes/cm, and the viscosity was 0.01 poise.

All the cases considered had a difference frequency of 400 Hz. Two bubbles were used. (1) A 20  $\mu\text{m}$  bubble with a resonance of 183 kHz is illustrative of the small bubble, high frequency resonance case. (2) A 500  $\mu\text{m}$  bubble (7.13 kHz resonance) was used with two sets of driving frequencies: one where the frequencies are on the order of the resonance frequency (4.3 kHz and 4.7 kHz) and the other where the frequencies are above the resonance frequency (15.8 kHz and 16.2 kHz).

##### A. Small Bubbles ( $f_0 \gg f_1, f_2$ )

As an example of a small bubble case, a 20  $\mu\text{m}$  bubble driven at 4.3 kHz and 4.7 kHz is considered. In the present context "small" is taken to mean small enough that the resonance frequency (which is inversely proportional to the equilibrium radius) is much greater than the other frequencies. In this case, the  $f_0$  is 183 kHz and easily satisfies this criterion. It should be noted, however, that the surface tension contribution to the internal pressure at equilibrium is only 0.075 bars. Therefore this is still a very compliant bubble.

The Fourier transform of the normalized bubble radial motion,  $x$ , for a drive pressure of 0.1 bar per primary ( $\xi_1 = \xi_2 = 0.1$ ) is shown in Fig. 3. Here  $\xi_1 = P_1/P_\infty$ . The most obvious feature of this figure is the narrowness of the lines. In fact, the lines are narrower than a resolution cell and a true estimate of the line level must allow for the fact that part of the energy shows up in sidebands for any finite FFT

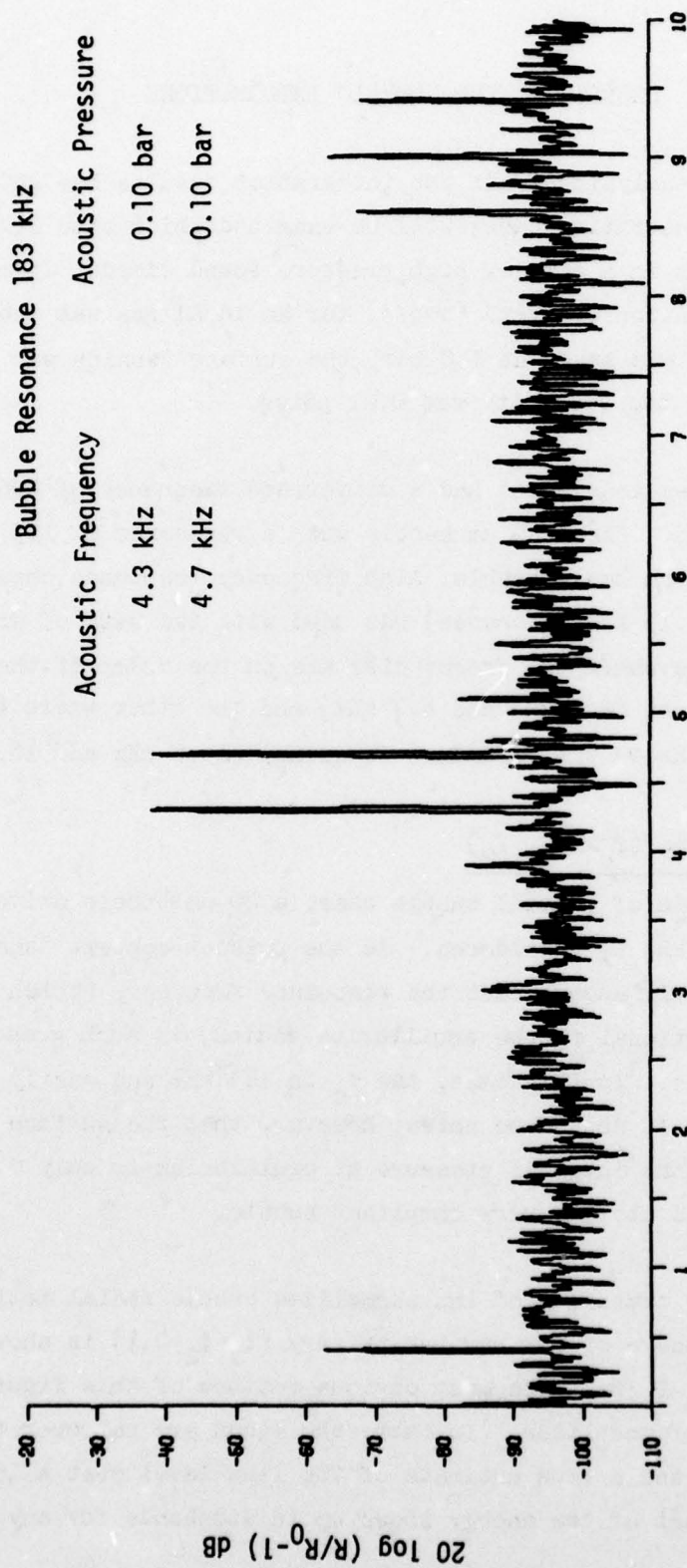


FIGURE 3

RADIAL MOTION SPECTRUM OF 20  $\mu$ m BUBBLE

if the line is not exactly in the cell center. A Hanning window has been applied to these transforms to minimize this effect, but it is still present.

The two lines of the drive frequencies appear to be at -37 dB from a measurement of the levels in Fig. 3. A more exact value using the adjacent cell values gives -34.4 dB, in excellent agreement with Eq. (10) which predicts -34.0 dB. In addition to the lines at the driving frequencies, second order lines at the difference frequency of 400 Hz and at frequencies of  $2f_1$ ,  $2f_2$ , and  $f_1+f_2$  are shown. As predicted by Eqs. (18) and (19) and discussed in section II, the energy in the latter three lines is divided evenly between the two second harmonics on the one hand and the sum frequency on the other. The spectrum of  $x$  on an expanded frequency scale is shown in Fig. 4. Here third order mixing effects show up as lines around  $3f_c$  and also as sidebands on the main line at the driving frequencies. The exact general perturbation solution for these lines has not been presented here, but in the limit of  $f_0 > f_1, f_2$ , the central line should be

$$60 \log(\xi) - 20 \log \left\{ (3\gamma) \left( \frac{3\gamma+1}{2} \right) \left( \frac{3\gamma+2}{3} \right) \right\} = -85 \text{ dB} ,$$

which appears to be in good agreement with Fig. 4.

A list of all first and second order line levels and the perturbation theory predictions is given in Table II for these driving frequencies and this bubble. Results for two higher driving pressures are also shown. The perturbation theory results are remarkably accurate at  $\xi$  of 0.25 but diverge from the integrated results at 0.5 bar driving pressure.

The spectrum of the 0.25 bar driving pressure is shown in Figs. 5 through 7. The higher order terms are present through many orders as shown in Fig. 6. It is interesting that the first and second order terms are predicted so accurately when so many higher orders have a significant amount of energy. Figure 7 shows this spectrum to a much higher frequency



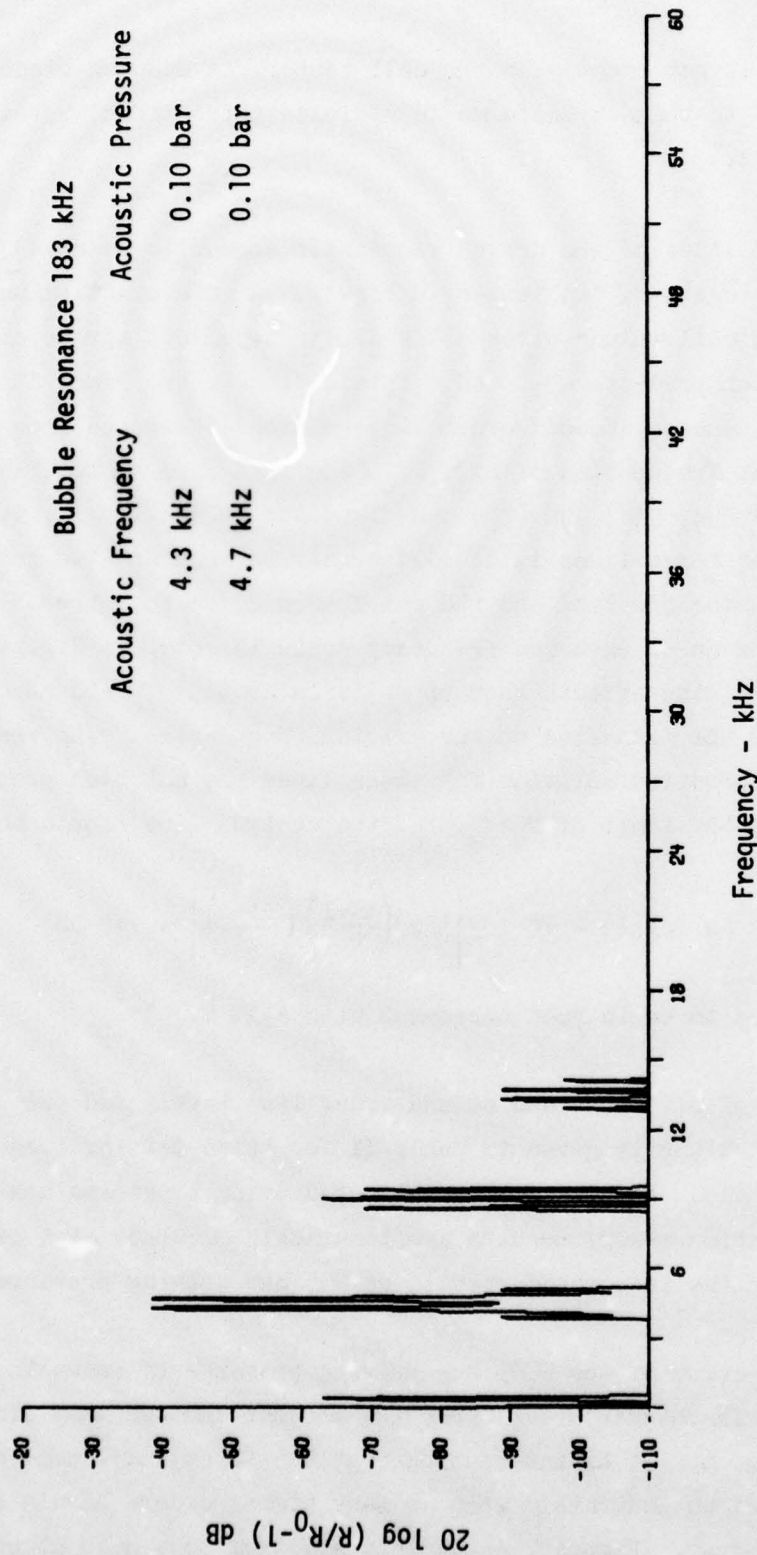


FIGURE 4

RADIAL MOTION SPECTRUM OF 20  $\mu$ m BUBBLE

TABLE II  
SMALL BUBBLE CASE  
COMPARISON OF INTEGRATION RESULTS AND PERTURBATION THEORY  
Values of x in decibels

Pressure	0.1 bar/primary			0.25 bar/primary			0.5 bar/primary		
	0	PT	0 - PT	0	PT	0 - PT	0	PT	0 - PT
Frequency									
$f_1$	-34.4	-34.0	-0.4	-26.0	-26.0	0.0	-16.7	-20.0	3.3
$f_2$	-34.4	-34.0	-0.4	-26.0	-26.0	0.0	-16.8	-20.0	3.2
$2f_1$	-65.2	-64.4	-0.8	-48.2	-48.4	0.2	-28.2	-36.4	8.2
$2f_2$	-65.2	-64.4	-0.8	-48.1	-48.4	0.3	-28.2	-36.4	8.2
$f_1 + f_2$	-59.2	-58.4	-0.8	-42.4	-42.4	0.0	-24.5	-30.4	5.9
$f_2 - f_1$	-59.2	-58.4	-0.8	-42.4	-42.4	0.0	-21.2	-30.4	9.2

$R_0 = 20 \mu\text{m}$        $f_0 = 183 \text{ kHz}$

$f_1 = 4.3 \text{ kHz}$        $f_2 = 4.7 \text{ kHz}$

0 = Integrated Results

PT = Perturbation Theory

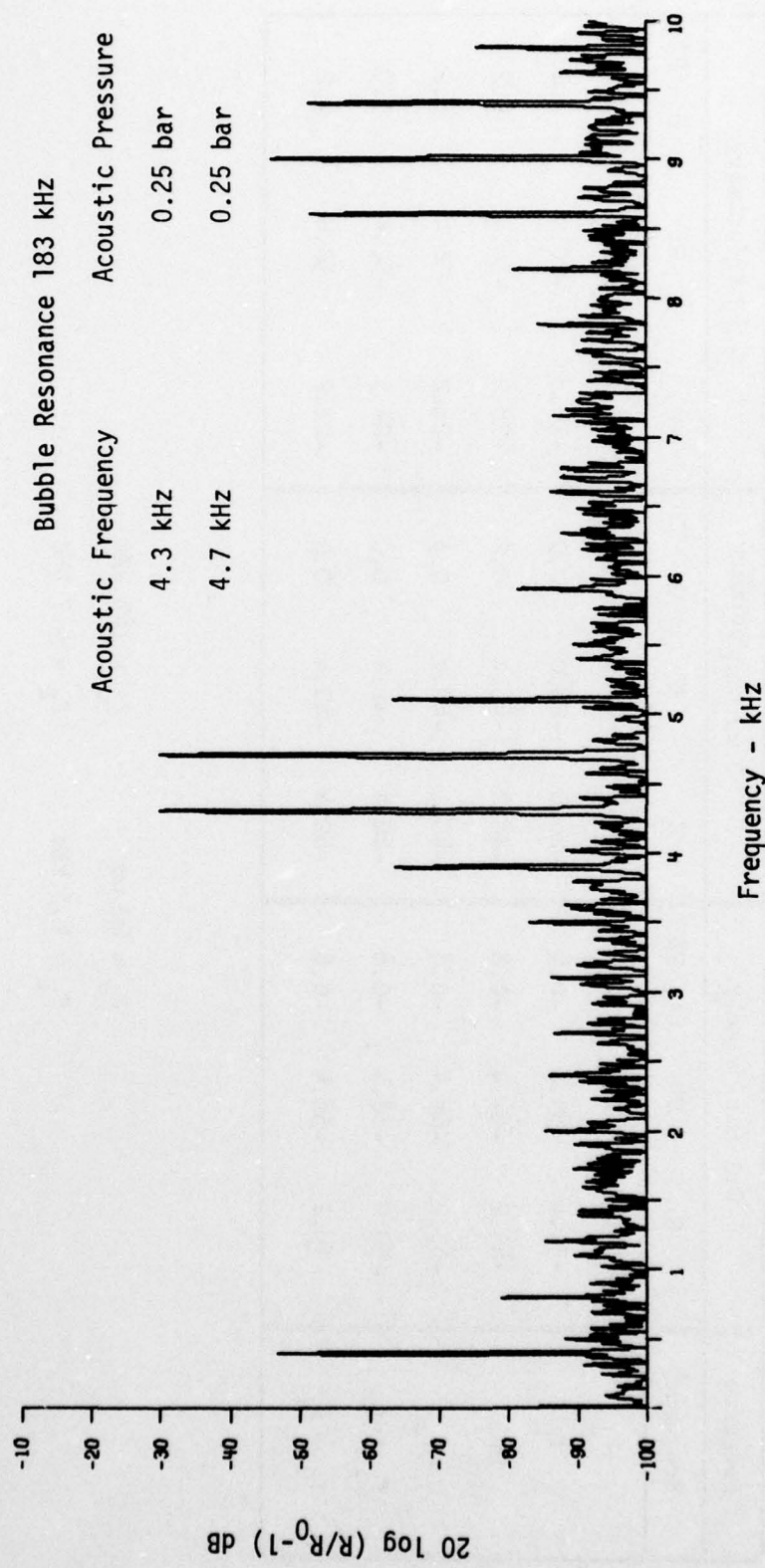


FIGURE 5  
RADIAL MOTION SPECTRUM OF 20 μm BUBBLE



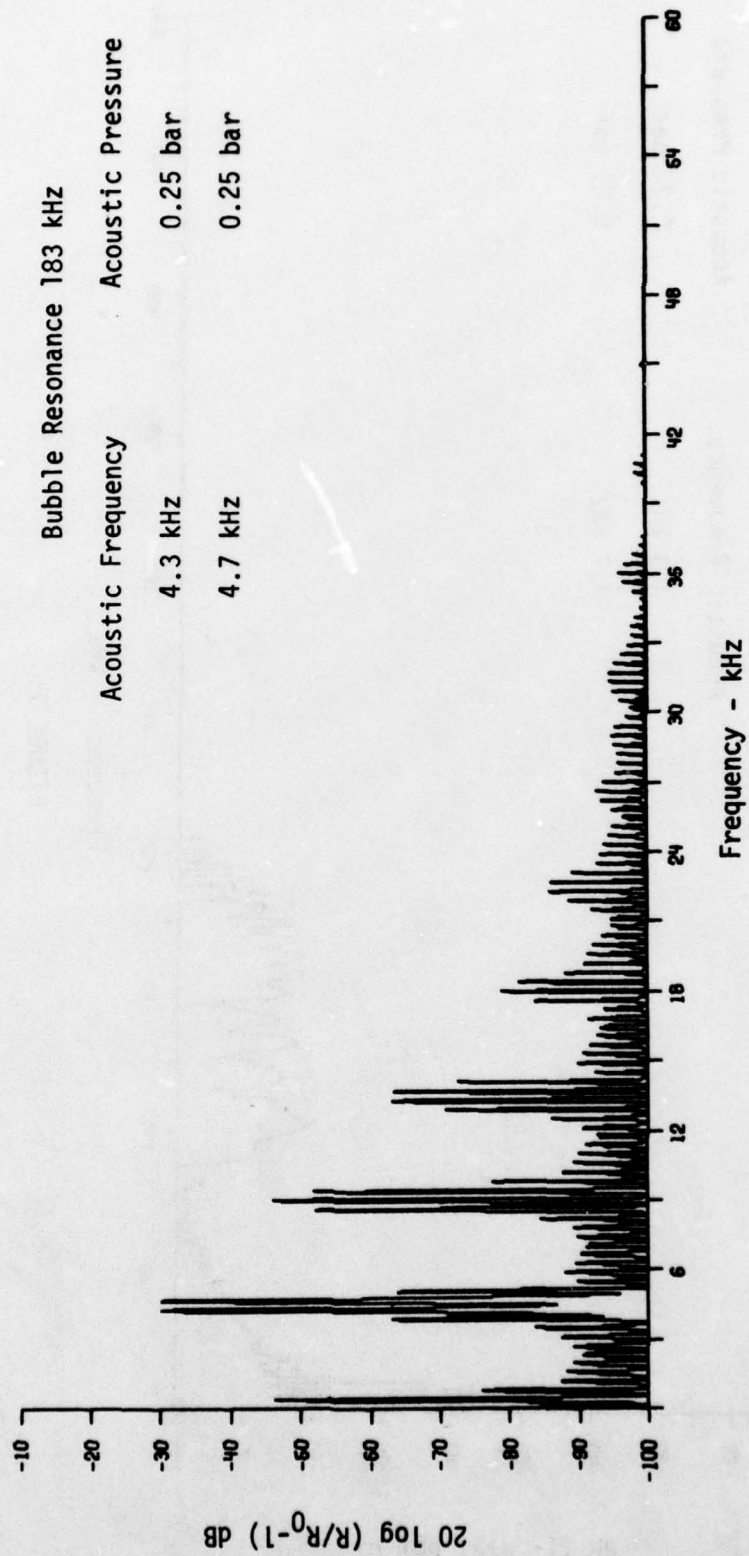


FIGURE 6  
RADIAL MOTION SPECTRUM OF 20  $\mu$ m BUBBLE

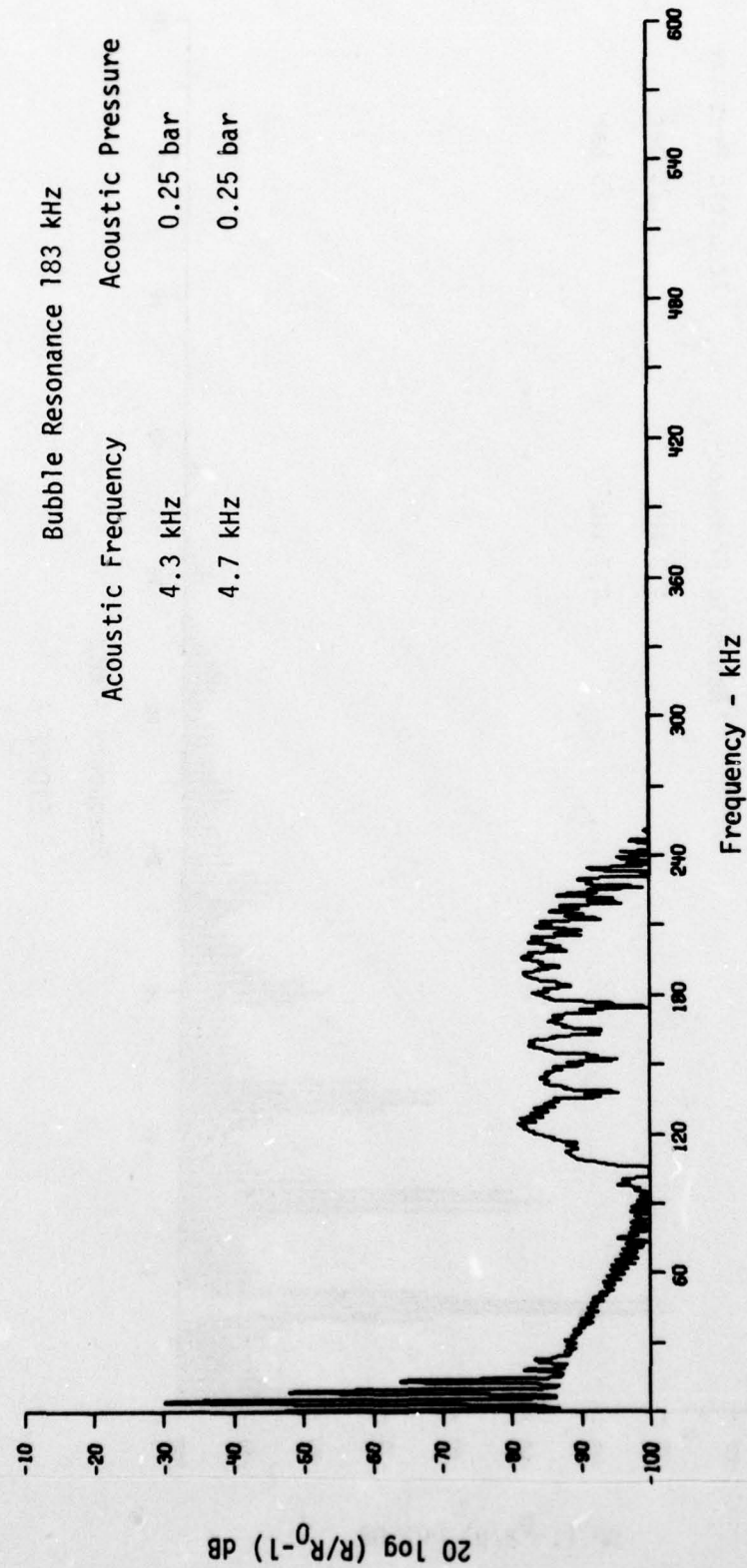


FIGURE 7  
RADIAL MOTION SPECTRUM OF 20  $\mu$ m BUBBLE

including the region of the bubble resonance. In this case a band of energy about the natural resonance frequency exists, but no narrow line is present. The first peak in this band is at 106 kHz while the last peak at the same power level is at 223 kHz. This broadband feature is caused by the slow variation of the driving frequency pressure being seen as a variation in the ambient pressure by the 20  $\mu$ m bubble whose resonance frequency is 183 kHz at 1 bar ambient pressure. This effect will be discussed later in section IV.D.

In Figs. 8 through 10 the spectrum of the 20  $\mu$ m bubble driven at 0.5 bar per primary is shown. The number of lines is considerable, but all these "low frequency" lines are still sharp in Fig. 8. Figures 9 and 10 show that the lines form "beat groups", separated by 4.5 kHz, which finally merge into a general background level. This general background extends to well beyond the natural resonance frequency. Again the natural resonance has been spread by an effective variation in ambient pressure.

At even higher driving pressures the broadband energy from bubble collapses overwhelms all other effects and even the driving frequencies are masked. This is evident in Figs. 11 and 12, which show spectra of this bubble driven with 1.0 bar per primary signals. In Table II, the lines at  $f_1$  and  $f_2$  were at -16.7 dB at the 0.5 bar driving level. If increased by only 6 dB in going to 1.0 bar (as predicted by perturbation theory), they would be at -10 dB, which is 15 dB below the noise level in the vicinity of  $f_1$  and  $f_2$ . The origin of the broad low frequency peaks around 550 Hz, 1100 Hz, 1200 Hz, etc., is unknown.

#### B. Medium Size Bubbles ( $f_0 > f_1, f_2$ ; $f_0 < f_1 + f_2$ )

To illustrate the intermediate case a 500  $\mu$ m bubble driven with the same 4.3 kHz and 4.7 kHz driving frequencies was chosen. The natural resonance frequency of the 500  $\mu$ m bubble is 7.13 kHz.



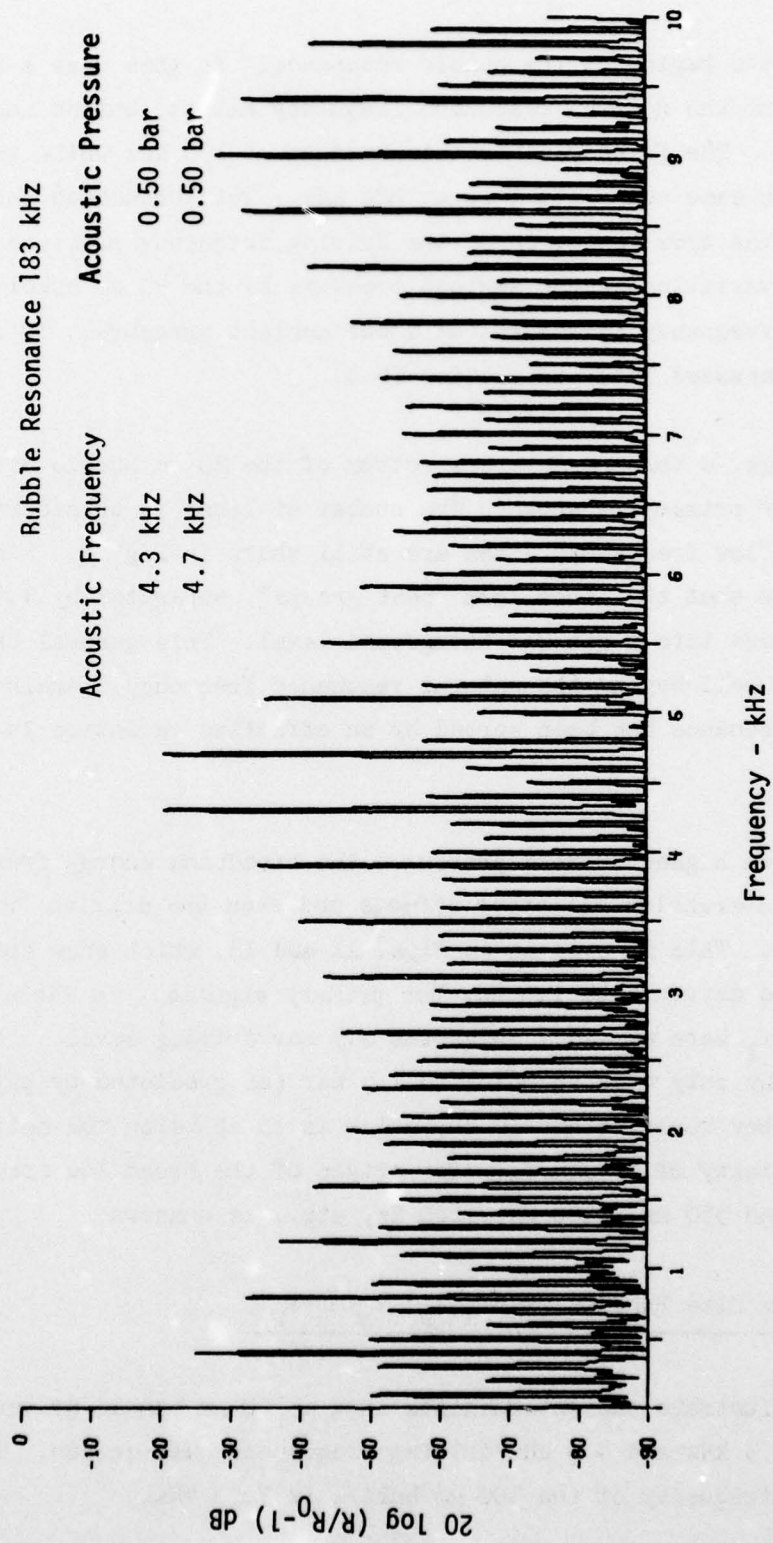


FIGURE 8

RADIAL MOTION SPECTRUM OF 20 μm BUBBLE

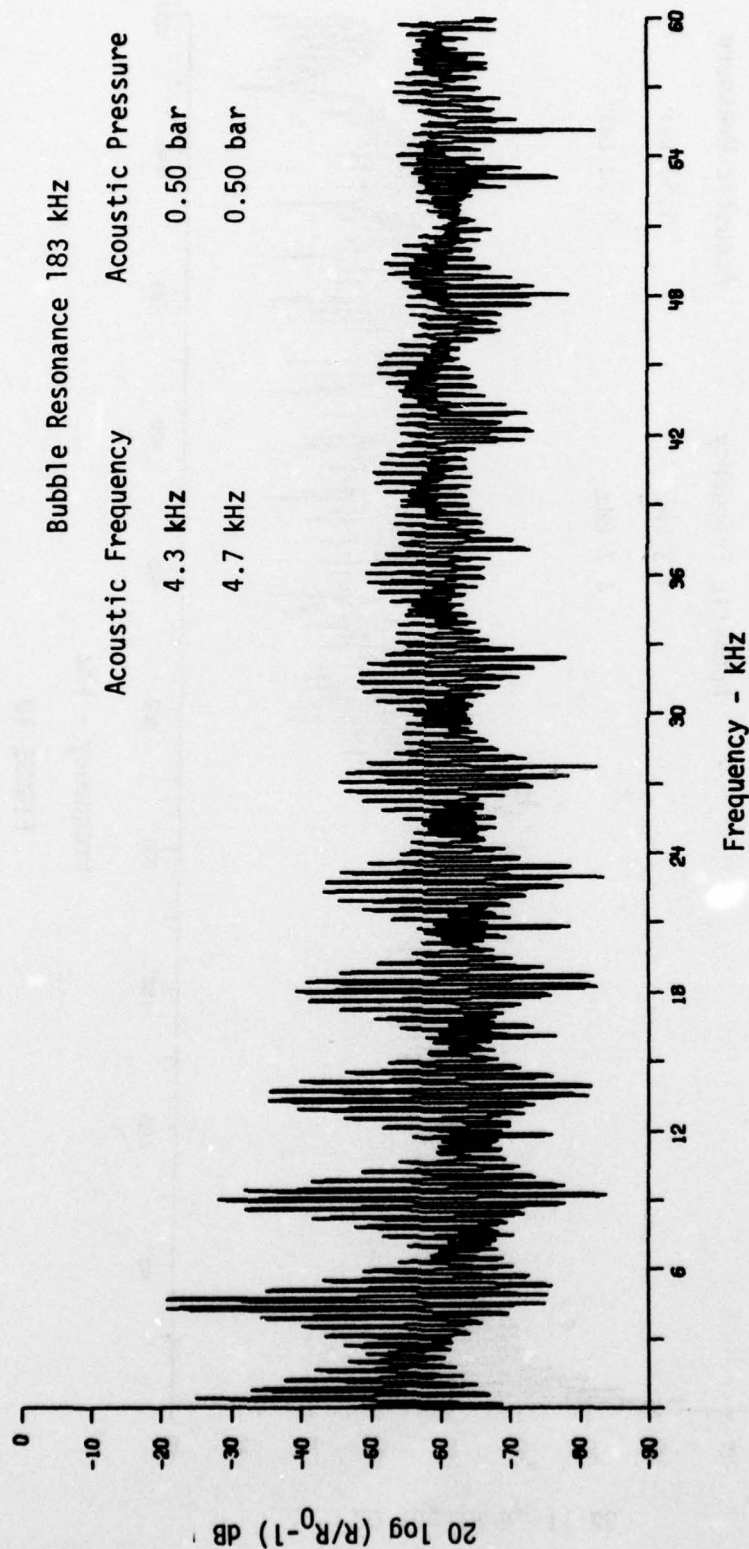
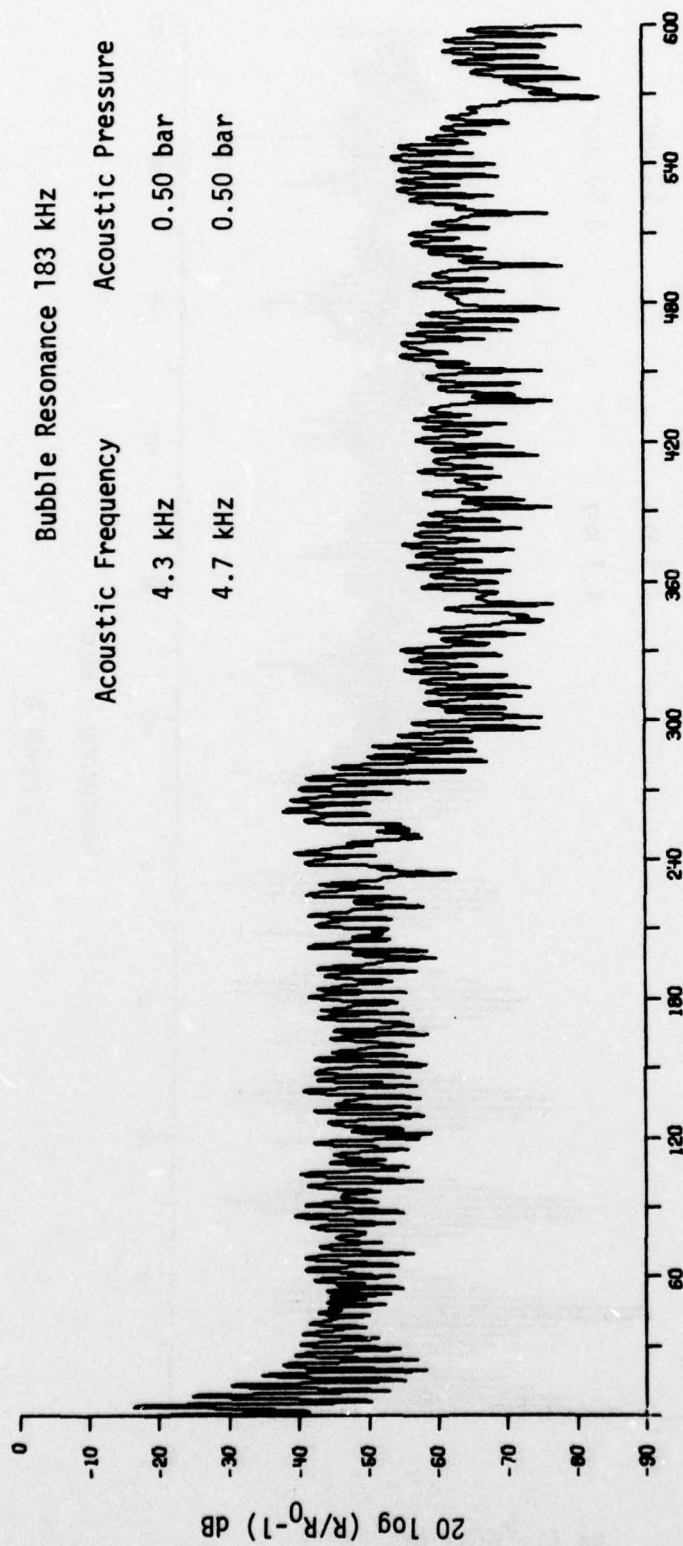


FIGURE 9  
RADIAL MOTION SPECTRUM OF 20  $\mu$ m BUBBLE



Frequency - kHz

FIGURE 10

RADIAL MOTION SPECTRUM OF 20  $\mu$ m BUBBLE



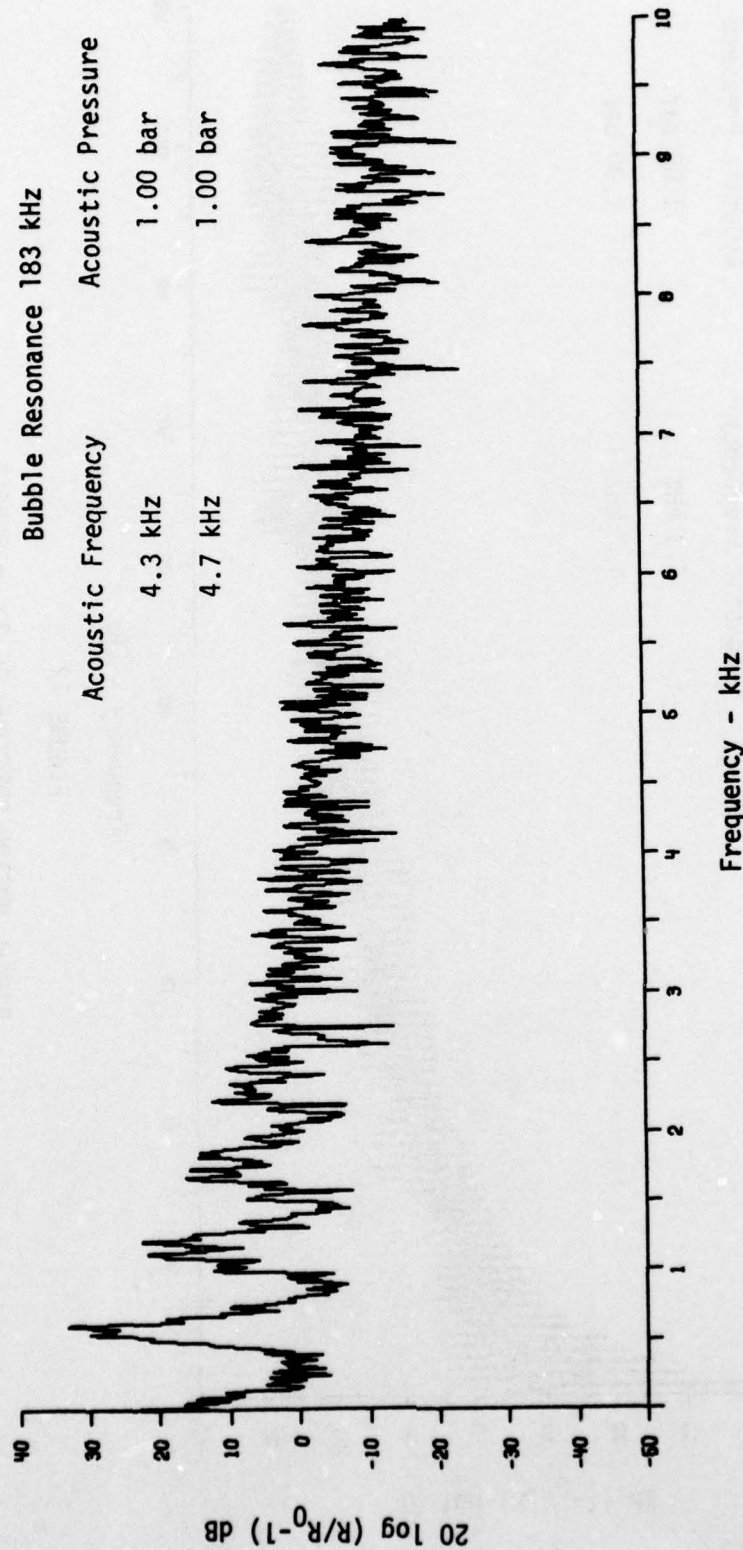


FIGURE 11  
RADIAL MOTION SPECTRUM OF 20  $\mu$ m BUBBLE

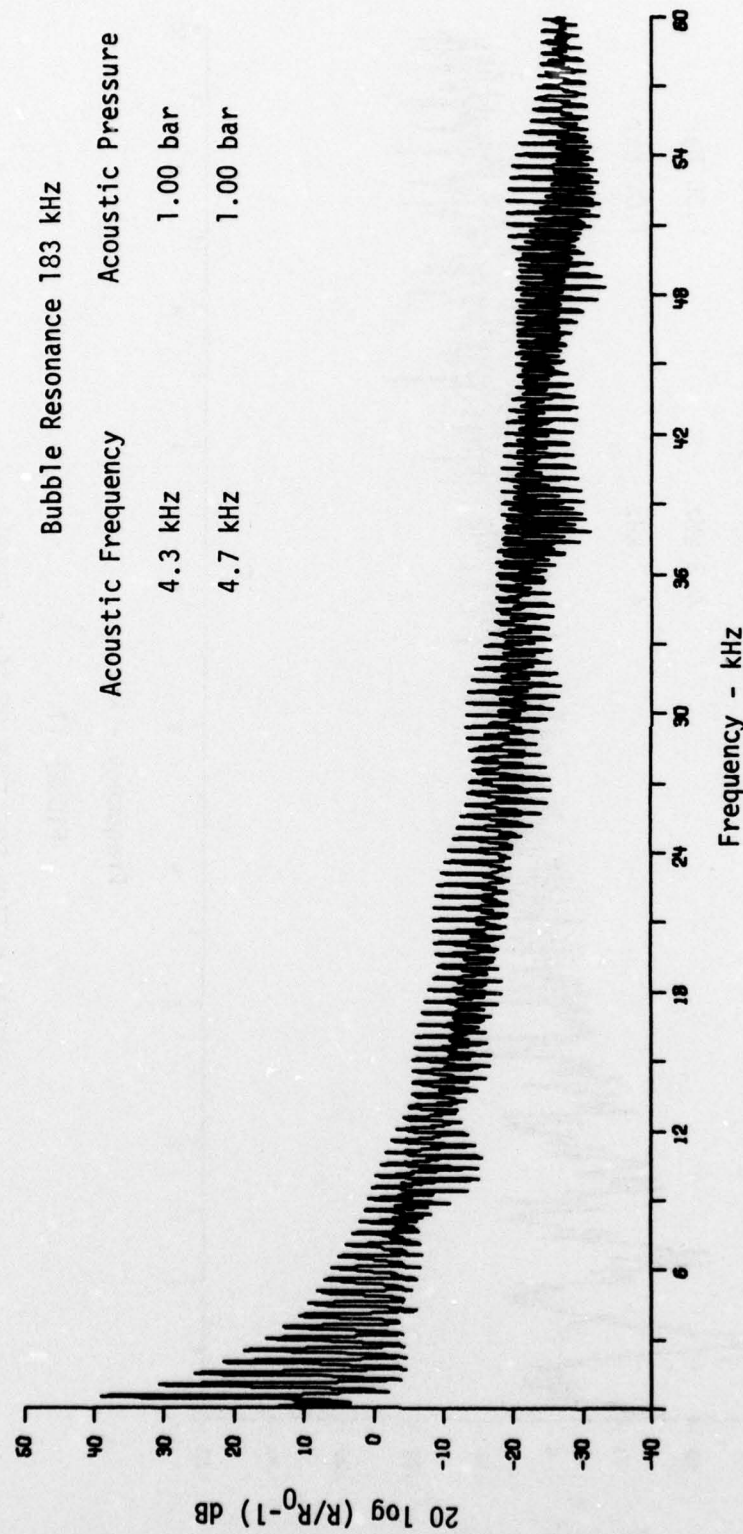


FIGURE 12  
RADIAL MOTION SPECTRUM OF 20  $\mu$ m BUBBLE

The spectrum of the bubble radial motion when the driving pressure is 0.1 bar per primary is shown in Fig. 13. In this case all lines are narrow, including the resonance line at 7.13 kHz. The third and higher order lines are also quite prominent even at this low driving pressure. Many of the frequency components on Fig. 13 have been labeled. The order of the term giving rise to the line is given in parentheses. Some fourth order lines show up on this graph.

Spectra for this case at driving pressures per primary of 0.25 bar, 0.50 bar, and 1.0 bar are shown in Figs. 14 through 16, respectively. At 0.25 bar, the general noise level is increasing, but many lines can still be distinguished. At 0.5 bar and 1.0 bar, all lines are masked by the general broadband noise. The dip formed by the noise decreasing from 0 Hz and the increased noise from the broadband (at 1.2 kHz on Fig. 14) was seen in many cases at high driving levels. An interesting feature of Fig. 16 is its similarity to Fig. 11. The absolute scales are also approximately the same when the normalization by  $R_0$  is removed. This implies that, at high driving pressures in the two-frequency case, the bubble spectrum becomes approximately independent of the original bubble size. The period between successive pressure absolute minimum, which is equal to the difference frequency period, apparently determines the maximum size to which the bubble can grow. If this explanation is correct, the spectra would not follow this pattern when  $f_2 - f_1$  approached the resonance frequency.

A summary of the line levels measurable in the 0.1 bar and 0.25 bar cases is shown in Table III. As with the small bubble case, perturbation theory predicted the 0.1 bar values quite well. The predictions for the first order terms also held true at 0.25 bar but the second order terms are in error by up to 5.7 dB.



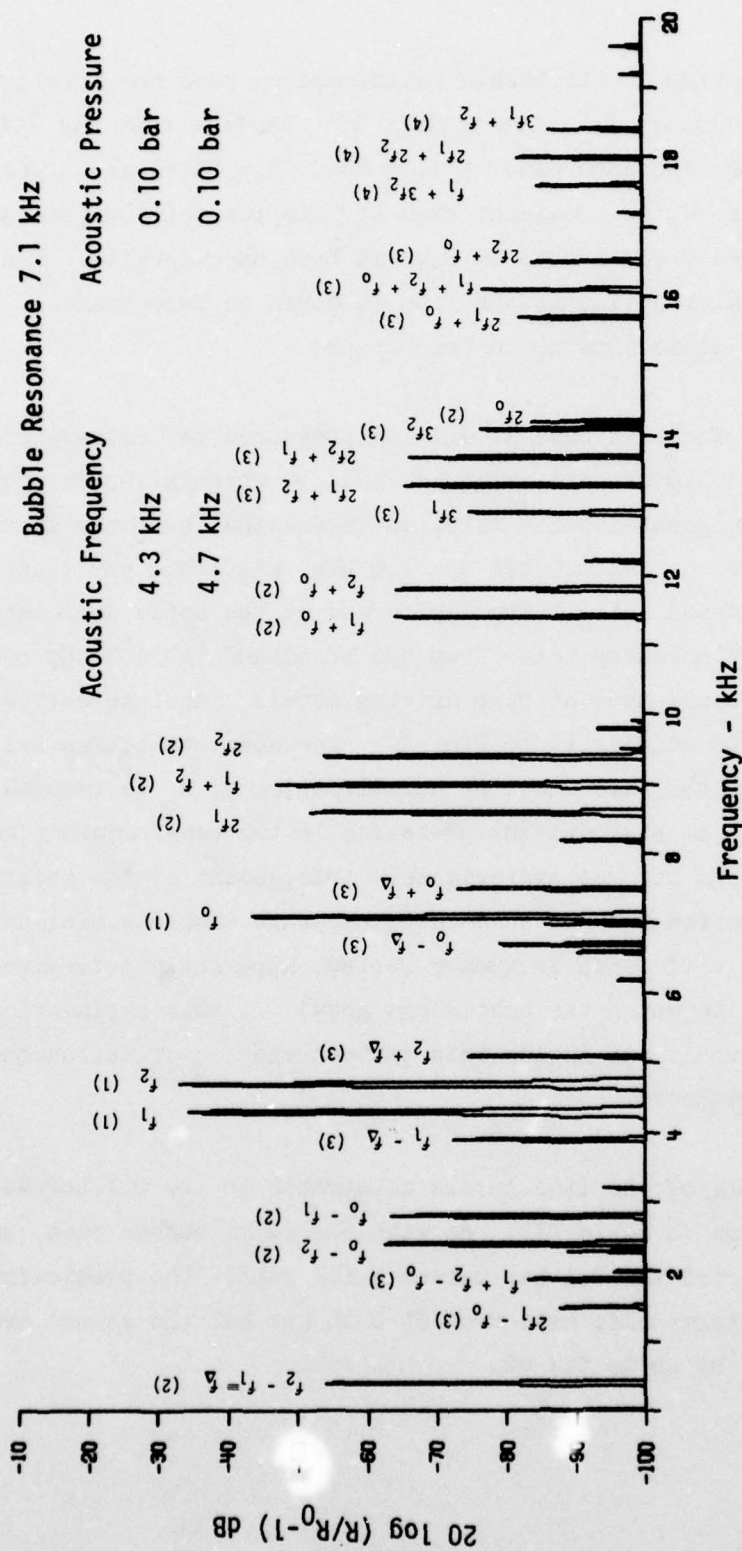


FIGURE 13  
RADIAL MOTION SPECTRUM OF 500  $\mu$ m BUBBLE

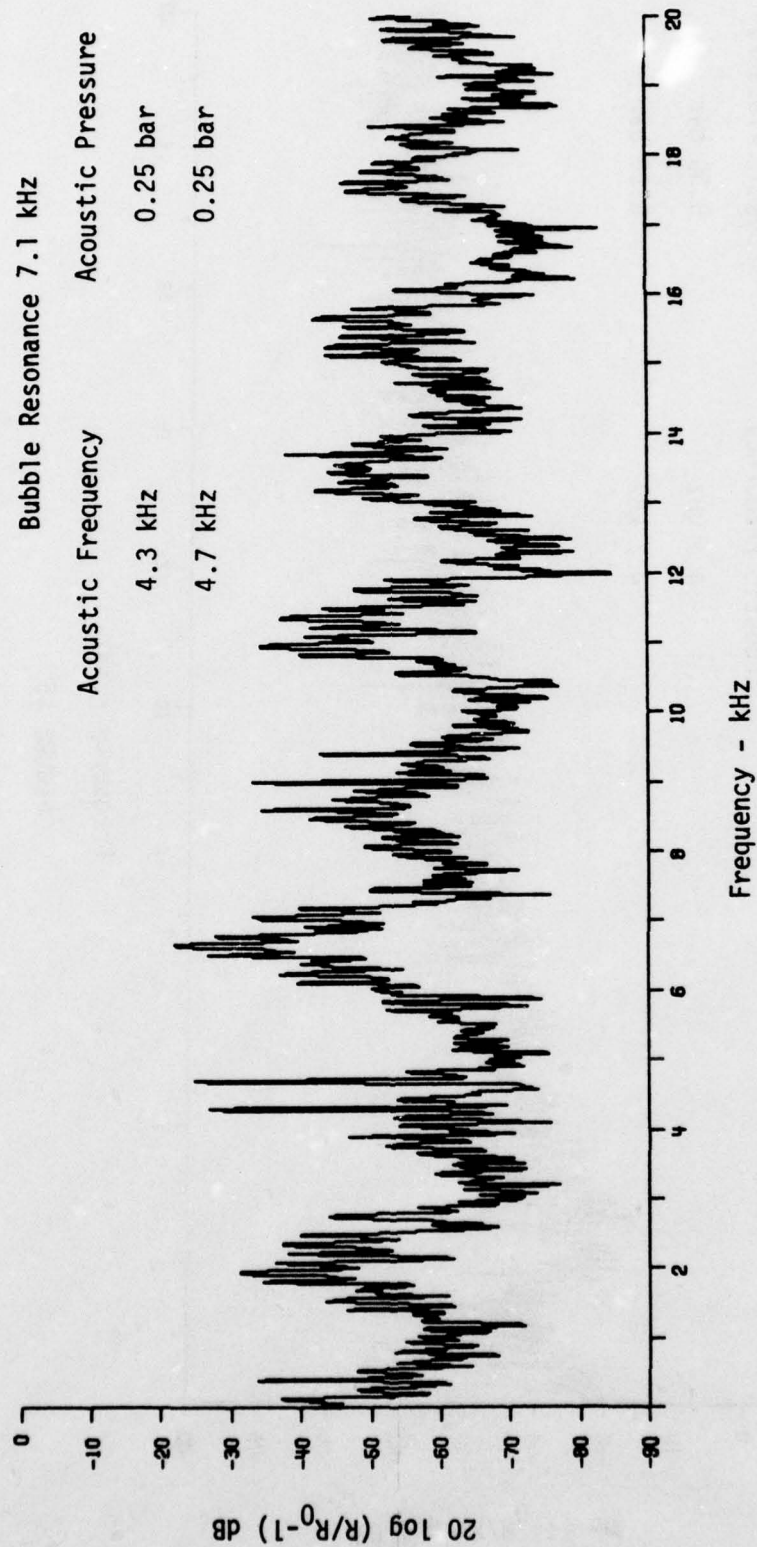


FIGURE 14  
RADIAL MOTION SPECTRUM OF 500  $\mu$ m BUBBLE

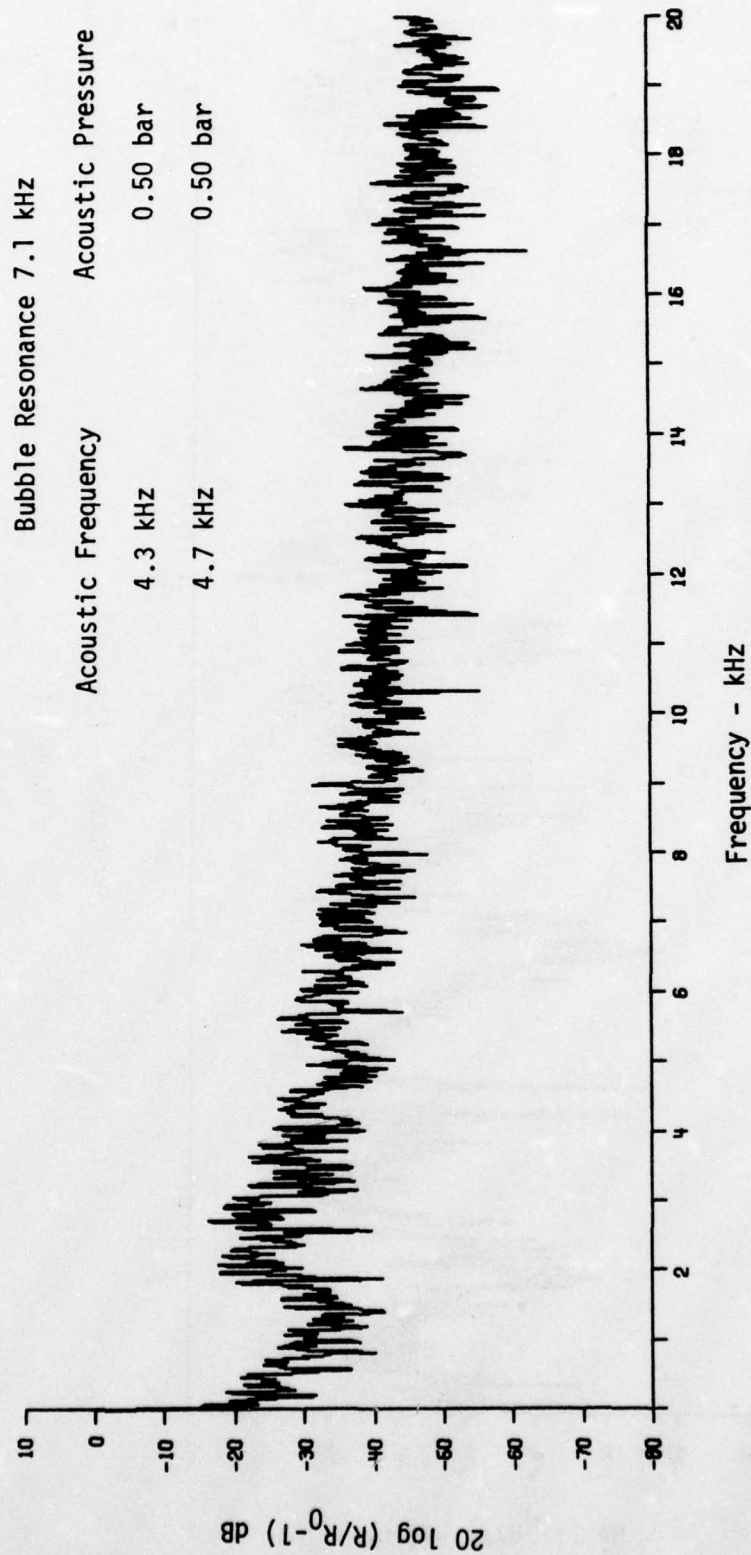


FIGURE 15  
RADIAL MOTION SPECTRUM OF 500  $\mu$ m BUBBLE



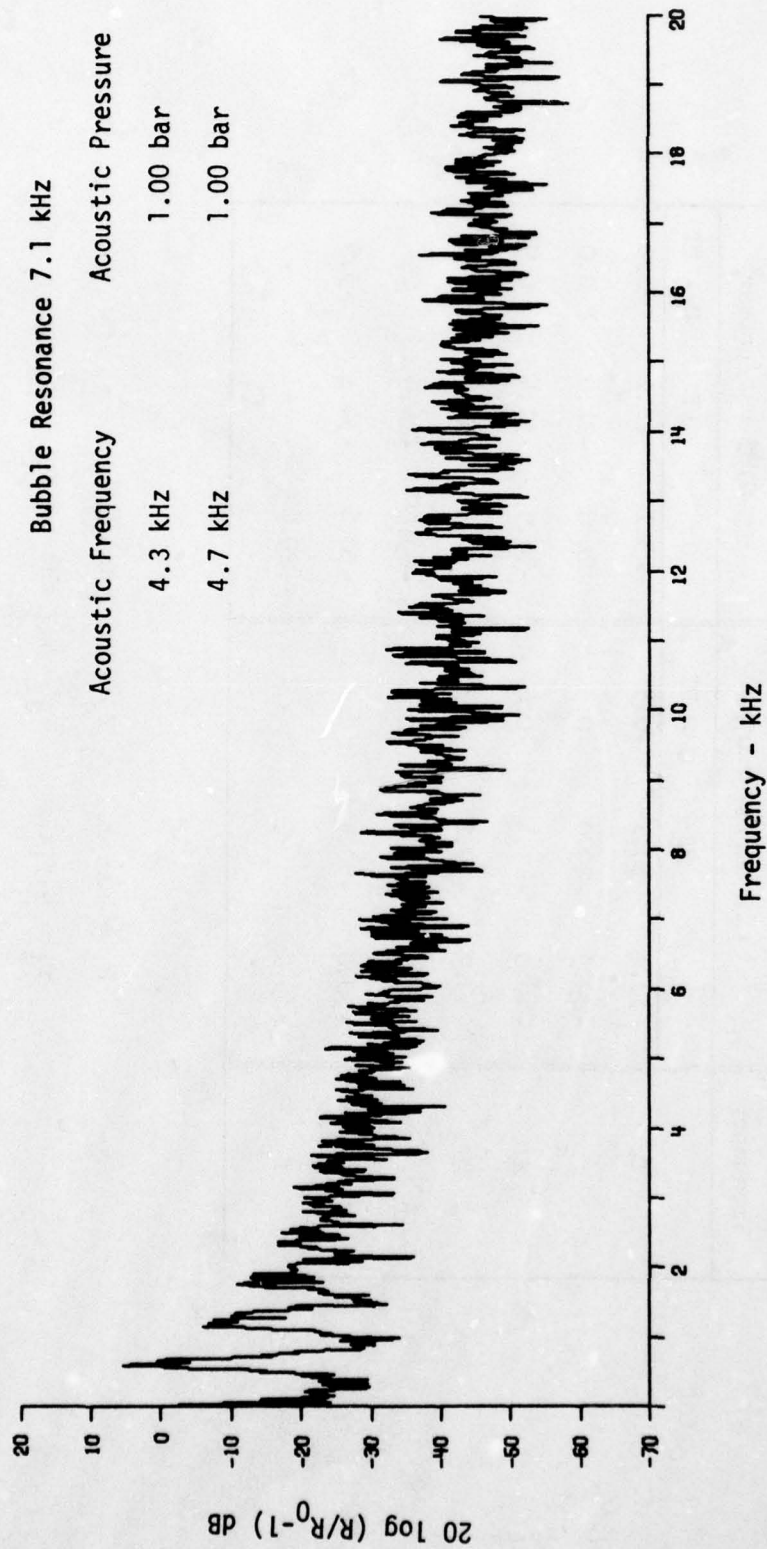


FIGURE 16  
RADIAL MOTION SPECTRUM OF 500  $\mu$ m BUBBLE

TABLE III  
NEAR-RESONANCE CASE  
COMPARISON OF INTEGRATION RESULTS AND PERTURBATION THEORY  
Values of x in decibels

Frequency	0.1 bar/primary			0.25 bar/primary		
	0	PT	0 - PT	0	PT	0 - PT
$f_1$	-30.4	-30.1	-0.3	-23.1	-22.1	-1.0
$f_2$	-29.3	-29.0	-0.3	-21.0	-21.0	0.0
$2f_1$	-48.0	-47.4	-0.6	-30.5	-31.4	0.9
$2f_2$	-49.7	-49.2	-0.5	-38.9	-33.2	5.7
$f_1 + f_2$	-42.9	-42.9	0.0	-29.3	-26.9	2.4
$f_2 - f_1$	-50.0	-50.1	0.1	-30.6	-34.1	-3.5
$f_0$	-39.5	-	-	-29.6	-	-

$R_0 = 500 \mu m$        $f_0 = 7.13 \text{ kHz}$

$f_1 = 4.3 \text{ kHz}$        $f_2 = 4.7 \text{ kHz}$

0 = Integrated Results  
PT = Perturbation Theory

### C. Large Bubbles ( $f_0 < f_1, f_2$ )

The large bubble regime was investigated using the same 500  $\mu\text{m}$  bubble described in section IV.B. with driving frequencies of 15.8 kHz and 16.2 kHz. The difference frequency is therefore the same as in the two previous sections.

Figure 17 shows the radial motion spectrum of the 500  $\mu\text{m}$  bubble. The same features shown for the medium bubble size in Fig. 13 are apparent in Fig. 17. The first and second order lines are narrow at the levels predicted by perturbation theory, as shown in Table IV.

The spectra of the same bubble driven with frequencies of 15.8 kHz and 16.2 kHz at pressures of 0.25 bar, 0.50 bar, 1.0 bar, and 1.5 bar are shown in Figs. 18 through 21, respectively. The levels of the lines can be followed to the 1.0 bar driving pressure level and are given in Table IV. Only at the 1.5 bar level does the broadband spectrum emerge, and even then, the driving frequencies and resonance frequency lines are evident.

The perturbation theory values are in good agreement through driving pressures of 0.5 bar/primary with the exception of the  $f_1 + f_2$  line, which is 1.5 dB high even at 0.1 bar, and the difference frequency line, which is beginning to grow rapidly at this level. One very positive result is that the difference frequency line is much larger than perturbation theory predicts. At 1.0 bar it is about 20 dB higher than expected. Although its precise value was not computed by the program, the 400 Hz line is still evident above the noise in Fig. 21 and probably has a value around -23 dB.

### D. Effective Resonance Frequency

In several of the previous figures (Fig. 7 for example) the resonance was spread out. This occurred when the resonance frequency was higher



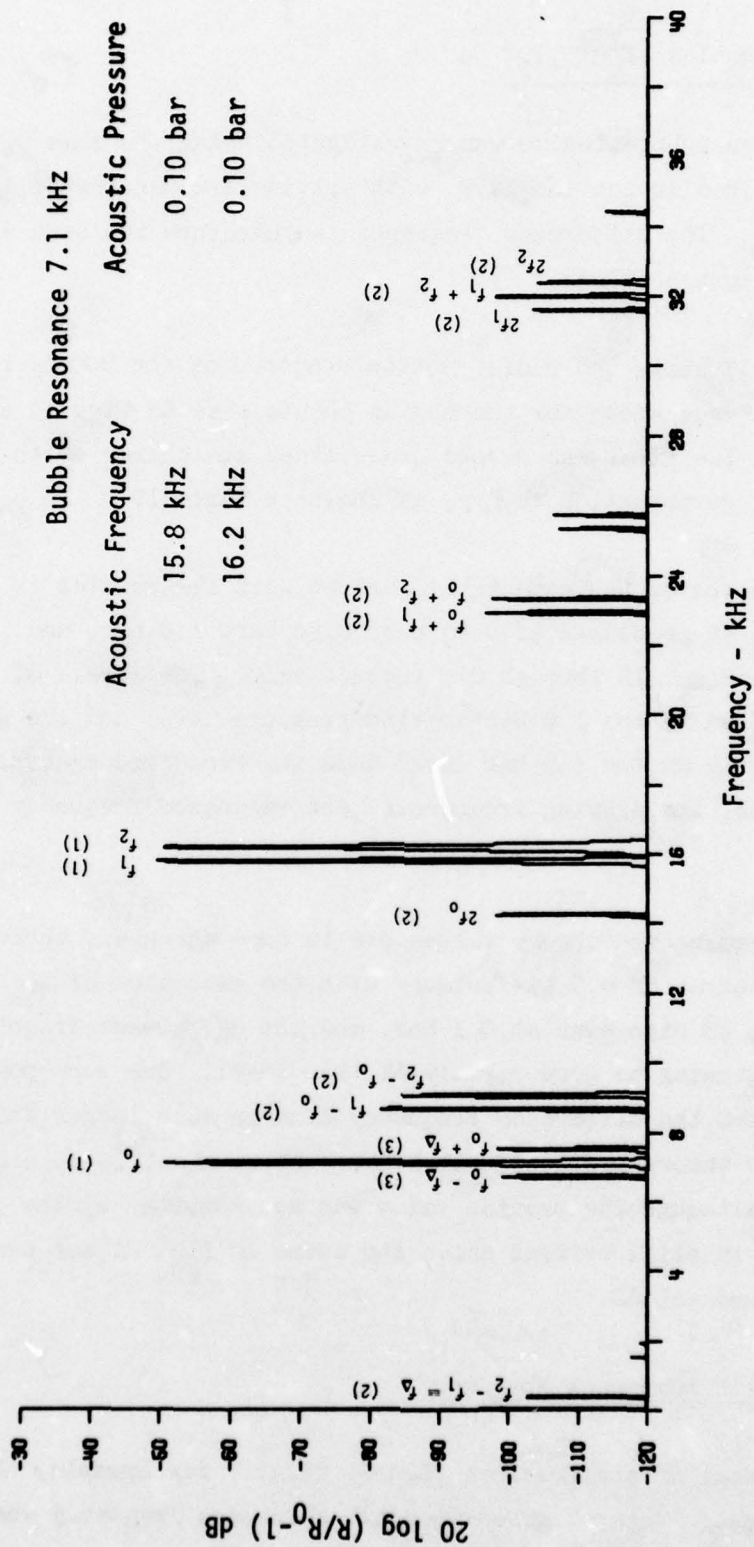


FIGURE 17  
RADIAL MOTION SPECTRUM OF 500  $\mu$ m BUBBLE

TABLE IV  
LARGE BUBBLE CASE  
COMPARISON OF INTEGRATION RESULTS AND PERTURBATION THEORY  
Values of  $x$  in decibels

P Frequency	0.1 bar/primary			0.25 bar/primary			0.5 bar/primary			1.0 bar/primary		
	0	PT	0 - PT	0	PT	0 - PT	0	PT	0 - PT	0	PT	0 - PT
$f_1$	-45.9	-45.8	-0.1	-37.9	-37.9	-0.1	-31.8	-31.8	0.0	-25.0	-25.8	0.8
$f_2$	-46.4	-46.4	0.0	-38.4	-38.4	0.0	-32.4	-32.4	0.0	-25.8	-26.4	0.6
$2f_1$	-99.5	-99.4	-0.1	-83.6	-83.4	-0.2	-71.4	-71.4	0.2	-53.6	-59.4	2.8
$2f_2$	-100.5	-100.6	-0.1	-84.7	-84.6	-0.1	-72.5	-72.6	0.1	-56.2	-60.6	4.4
$f_1 + f_2$	-94.0	-95.5	1.5	-78.1	-79.5	1.4	-65.9	-67.5	1.6	-49.9	-55.5	5.6
$f_2 - f_1$	-98.3	-98.5	0.2	-81.9	-82.5	0.4	-67.0	-70.5	3.5	-36.8	-58.5	21.7
$f_0$	-46.9	-	-	-38.6	-	-	-31.6	-	-	-20.2	-	-

$R_0 = 500 \mu\text{m}$      $f_0 = 7.13 \text{ kHz}$

$f_1 = 15.8 \text{ kHz}$      $f_2 = 16.2 \text{ kHz}$

0 - Integrated Results

PT - Perturbation Theory

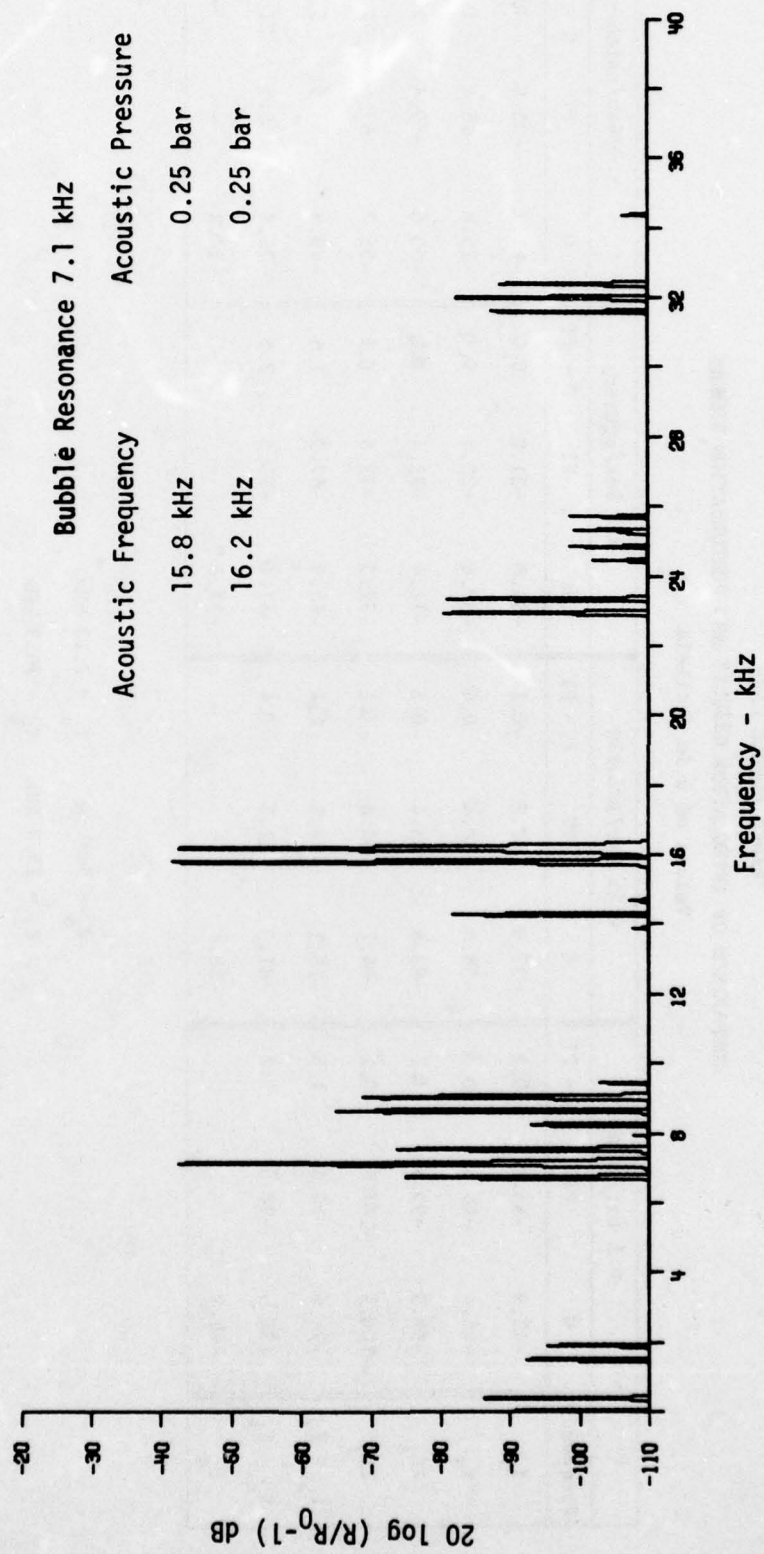


FIGURE 18

RADIAL MOTION SPECTRUM OF 500  $\mu$ m BUBBLE



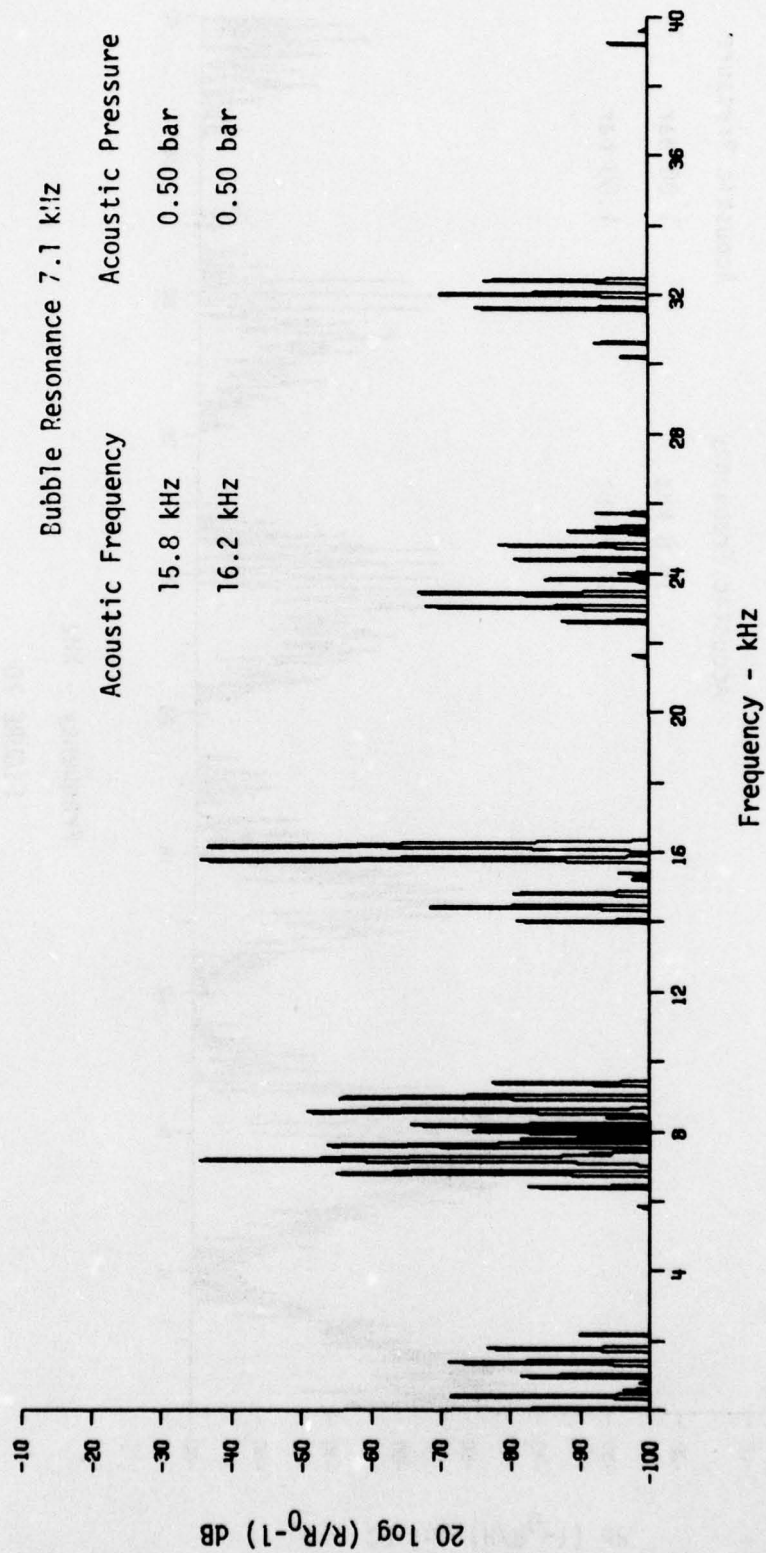
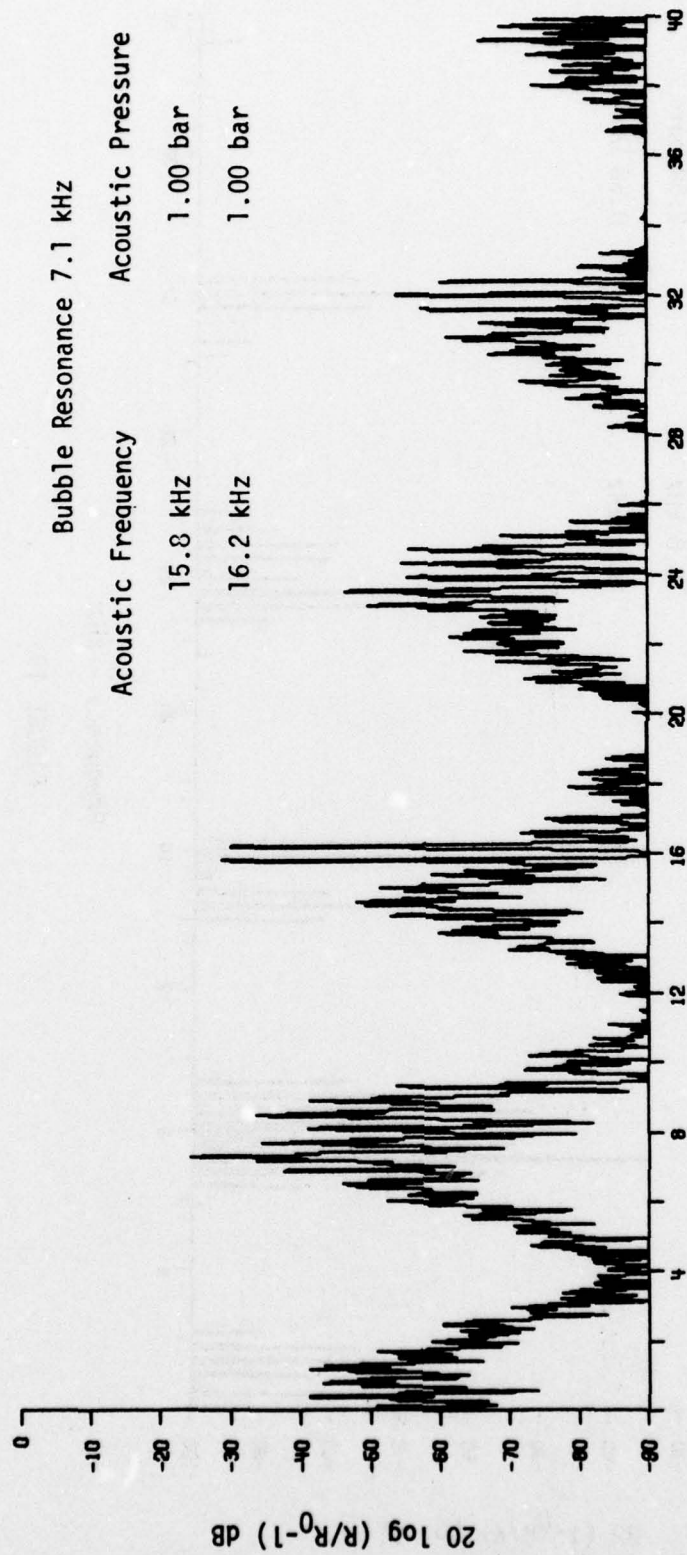


FIGURE 19

RADIAL MOTION SPECTRUM OF 500 μm BUBBLE



Frequency - kHz

FIGURE 20

RADIAL MOTION SPECTRUM OF 500  $\mu$ m BUBBLE

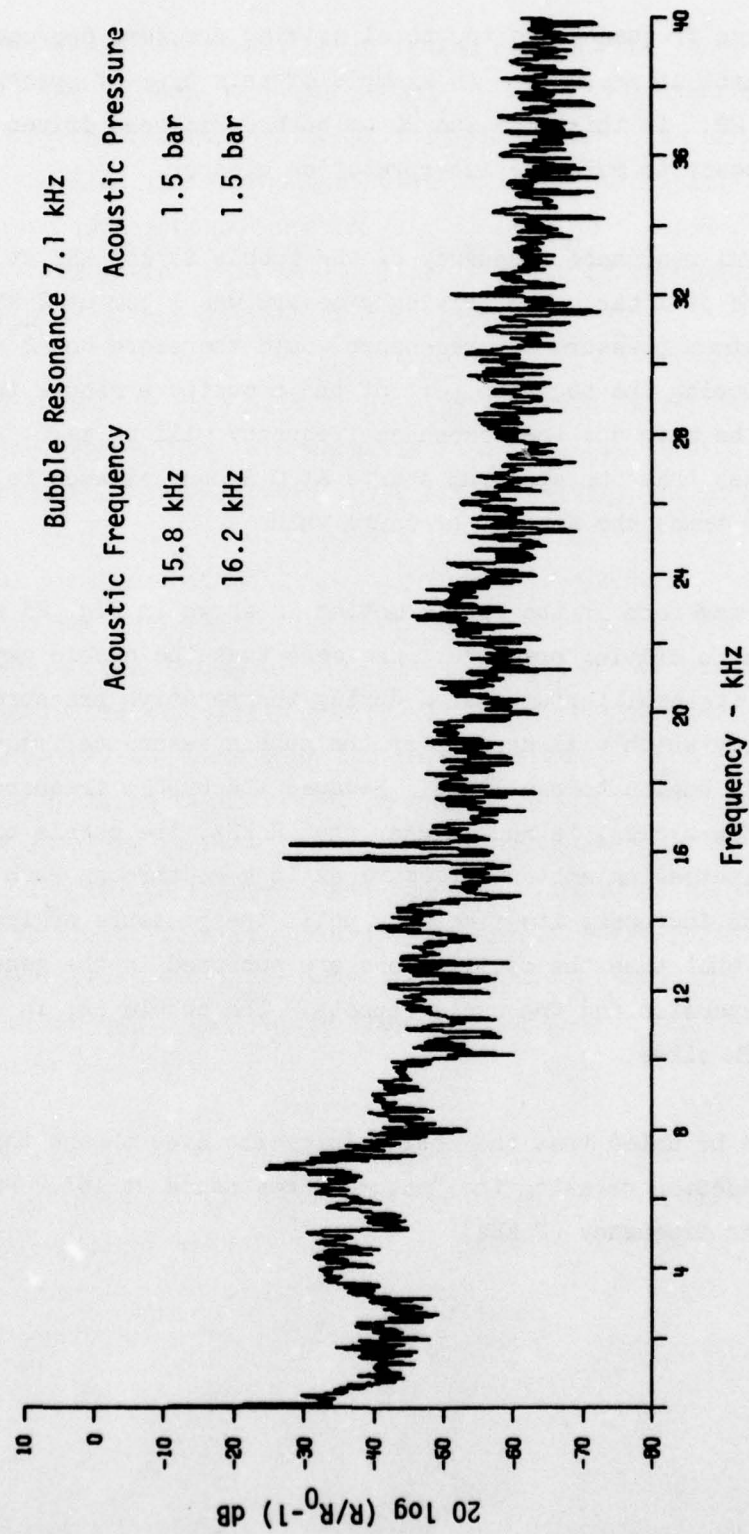


FIGURE 21  
RADIAL MOTION SPECTRUM OF 500  $\mu$ m BUBBLE



than the driving frequency and the total driving pressure approached or exceeded the ambient pressure. An example of this type of spectrum is shown in Fig. 22. In this case the 20  $\mu\text{m}$  bubble had been driven with only one frequency to make the interpretation clearer.

The natural resonance frequency of the bubble is 183 kHz at a static pressure of 1 bar. The driving pressure was 1 bar at 2 kHz. During the maximum pressure the resonance would therefore be  $\sqrt{2} \times 183$  kHz or 259 kHz. During the negative part of the acoustic pressure the total pressure will be zero and the resonance frequency will be zero. In Fig. 22 one sees that the spectrum starts at 0 Hz but extends to about 290 kHz or 1.6 times the ambient pressure value.

The time waveform of the bubble motion is shown in Fig. 23 along with the acoustic driving pressure. One sees that the bubble expands to many times its equilibrium radius during the negative pressure phase, acquiring energy which will show up at the bubble resonance frequency once the bubble begins to oscillate. Because the bubble frequency, even on the first few cycles, is much higher than 2 kHz, the bubble experiences a constantly increasing ambient pressure as it goes through each oscillation and increases its frequency until the pressure begins to decrease. At that time the oscillations are subsumed in the general bubble oscillation expansion and the cycle repeats. The bubble is, in effect, executing an FM slide.

It should be noted that the motion is cyclic even though there is no rational fraction relating the "natural" resonance at 183.2 kHz and the driving frequency (2 kHz).

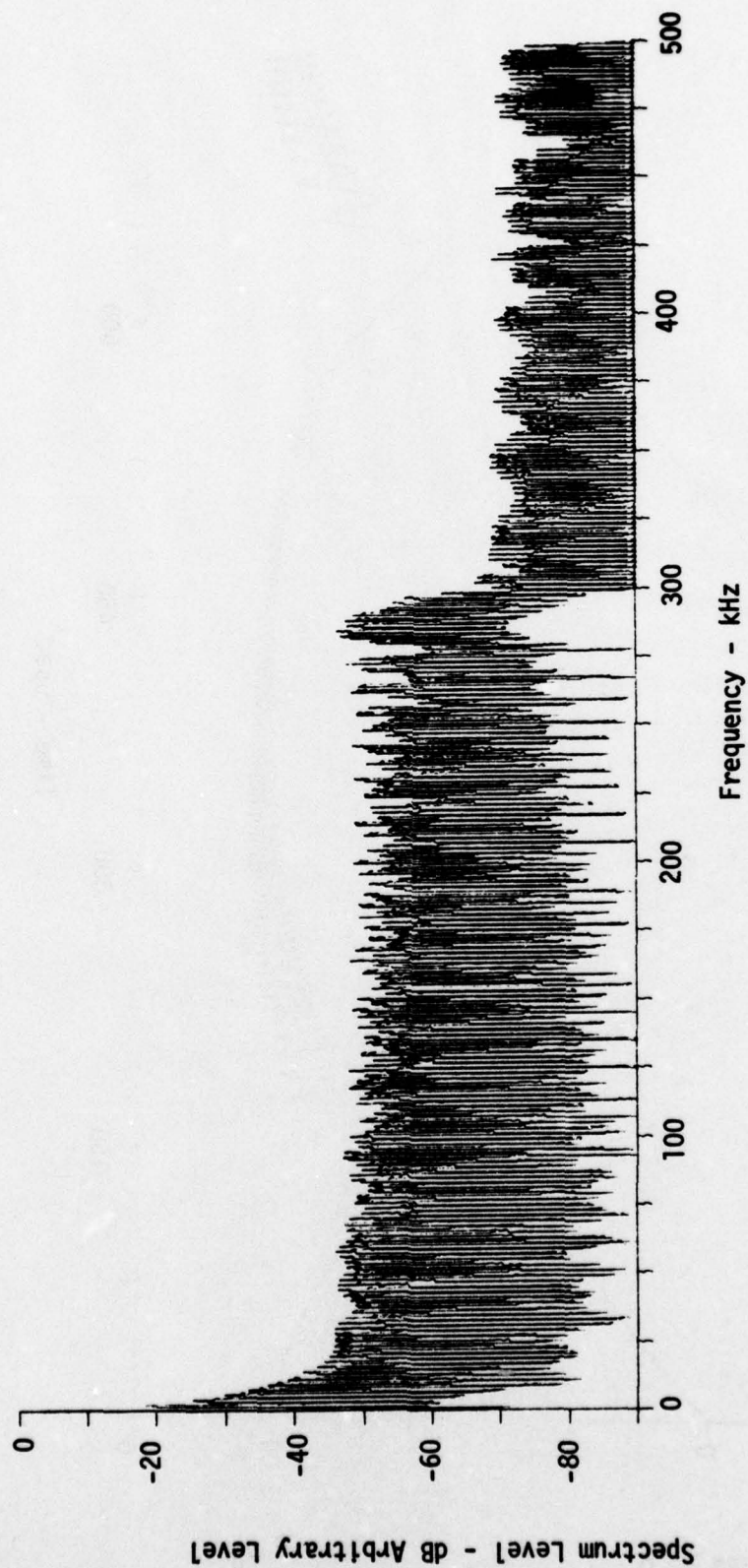


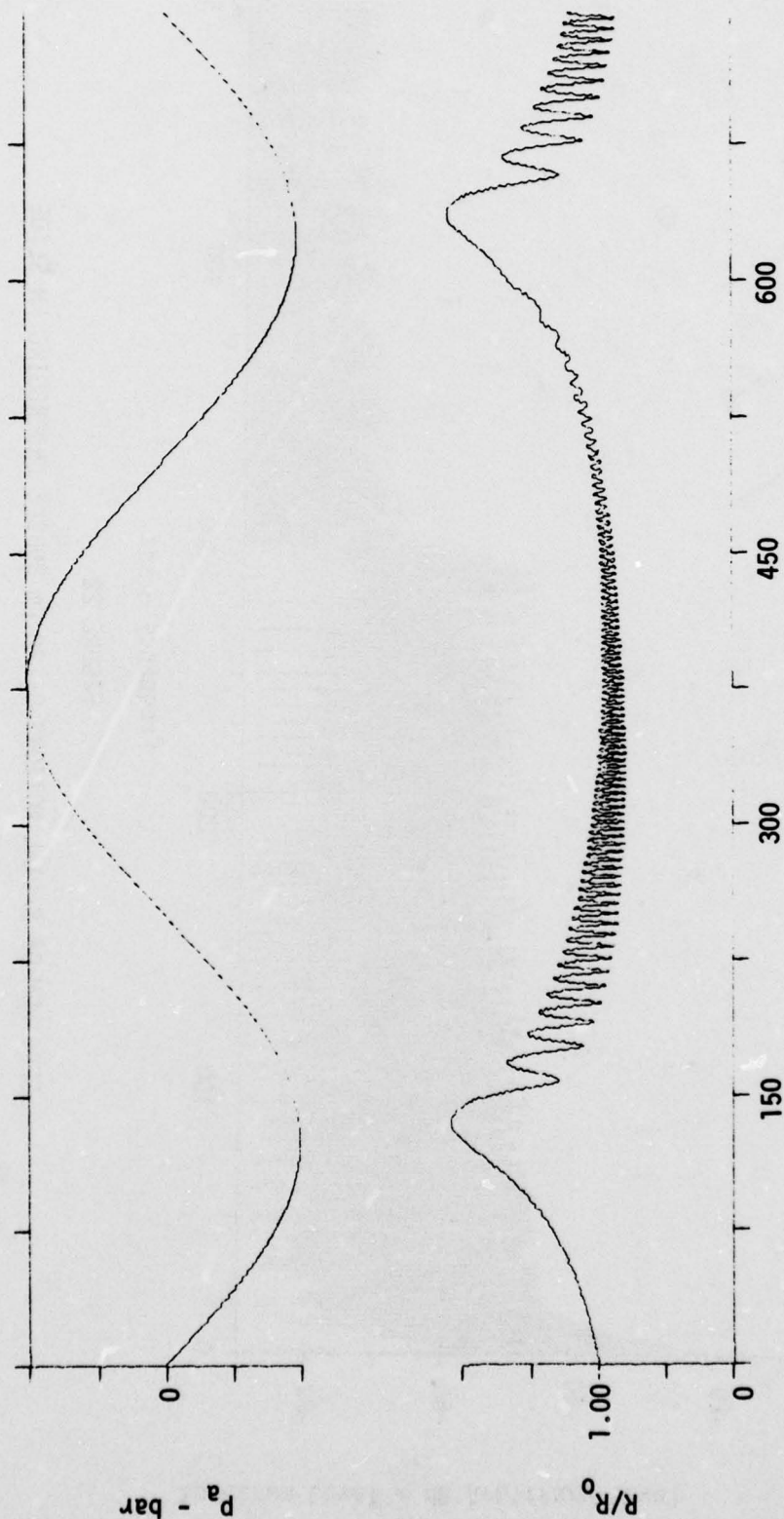
FIGURE 22

SPECTRUM OF RADIAL MOTIONS OF 20  $\mu$ m BUBBLE UNDERGOING FM SLIDE

AS-78-1397  
JRC 1095

Frequency  
2 kHz

Acoustic Driving Pressure  
1 bar



Time -  $\mu\text{sec}$

FIGURE 23

EXPANDED VIEW OF 20  $\mu\text{m}$  BUBBLE UNDERGOING FM SLIDE OSCILLATIONS



## V. SUMMARY

The equation of motion of a gas bubble in water has been numerically integrated for the case of being driven by two intense acoustic waves at frequencies  $f_1$  and  $f_2$ . Emphasis has been on the secondary frequency components generated by the nonlinear motion of the bubble, particularly the difference frequency  $f_2 - f_1$ . Three cases have been examined in detail: driving frequencies far below resonance (small bubble), driving frequencies near resonance, and driving frequencies above resonance (large bubble).

The numerical results were compared to results obtained by perturbation theory. Agreement was excellent at driving pressures up to one quarter of the ambient pressure in the small bubble case, while the near-resonance results had already begun to diverge at this level. The large bubble results were in good agreement with perturbation theory at half-ambient driving pressures except for the difference frequency component, which was 3.5 dB larger than predicted.

In all cases, at medium pressures a myriad of sharp lines were present, sometimes resulting from fourth order interactions. At high driving pressures (equal to or greater than ambient) sharp bubble collapses created broadband noise which obliterated the sharp lines. This indicates that driving pressures below the cavitation level may be more useful in generating difference frequency sound.

The rather large enhancement of difference frequency level over the perturbation theory solution in the large bubble case indicates that large bubbles are probably more useful than small ones. In addition the lines are present to much higher driving pressure levels. In designing

experiments and equipment the problem of scattering, which increases as the fourth power of the radius, must be considered and a detailed trade-off study would be required.

#### ACKNOWLEDGMENTS

The authors express their thanks to Werner Lauterborn who supplied an early version of the computer program. The version was designated GASBUBL and had extensively been modified by the time GASBUBL5 was operational. All errors are therefore the responsibility of the authors.



APPENDIX  
MAGNETIC TAPE FORMATS

#### A. GASBUBL5 Output

GASBUBL5 outputs four multiplexed channels of data to magnetic tape in SPIO format.\* These channels are time (T), bubble radius (R), bubble wall velocity ( $\dot{R}$ ), and bubble wall acceleration ( $\ddot{R}$ ) in that order. The data are grouped in units of 2048 points per array. Each of these units is called a "sequence" and consists of two physical records: an identification record (IDR) and a data record (DR). The IDR's are 121 CDC 3200 integer words long and the DR's are 8096 ( $4 \times 2048$ ) CDC 3200 integer words long. The records were written with BUFFER OUT in odd parity at 800 bpi. Each sequence takes 3.6 ft of tape allowing about 640 sequences to be placed on a 2400 ft tape.

Each IDR contains coded information to allow easy I/O using routines available at ARL:UT and information about the parameters used in the integration. Each IDR is equivalenced in GASBUBL5 to an integer (ID) and a floating point (XD) array. Because it requires two 24-bit CDC 3200 words to store one 48-bit floating point word, XD is only 60 elements long. The storage format of the IDR is given in Table AI.

The four multiplexed data channels have been scaled and stored as 24-bit integers prior to being placed on magnetic tape. The scale factors and other quantities necessary to retrieve the true values of the data are stored in the IDR. Because of automatic scaling within GASBUBL5, these scale factors may not be the same from sequence to sequence. To recover the data on a CDC 3200 computer, one must input the IDR and set

RZERO = XD(32),  
SCLT = XD(42),  
SLCR = XD(43),  
SCLRD = XD(44), and  
SCLRDD = XD(45).

\*A detailed explanation of this format is given in Ref. 18. However, the information in this appendix should be sufficient for a programmer to be able to read the tapes.

TABLE AI  
GASBUBL5 IDENTIFICATION RECORD

Integer Array

<u>ID Index</u>	<u>Storage Usage</u>
1	Number of elements in each array in following DR (2048).*
2	Sequence number, a sequential record index.
11	Initial suggested sample frequency (kHz)
19,21	Used by I/O routines.
91	Number of steps to back up in major restart (20).
93	Number of this sequence within current integration. If negative, indicates last sequence in integration.
94	Maximum number of consecutive step size reductions before doing a major restart (1).
100	Number of arrays multiplexed in DR (4).
121	Characters "SPIT", used to locate ID records.

Floating Point Array

<u>XD Index</u>	<u>Storage Usage</u>
22	$\gamma$ Adiabatic exponent
23	$\sigma$ Surface tension (dynes/cm)
24	$P_0$ Ambient pressure (bars)
25	$\mu$ Viscosity (poise)
26	$\rho_0$ Fluid density ( $\text{g/cm}^3$ )
27	$f_1$ First driving frequency (kHz)
28	$P_1$ Amplitude of pressure at frequency $f_1$ (bars)
29	$f_2$ Second driving frequency (kHz)
30	$P_2$ Amplitude of pressure at frequency $f_2$ (bars)
31	$\phi$ Beginning phase difference in driving amplitudes (radians)
32	$R_0$ Equilibrium radius ( $\mu\text{m}$ )
33	$P_v$ Vapor pressure inside bubble (bars)
34	$t$ Initial time for this integration ( $\mu\text{sec}$ )
35	$R$ Initial bubble radius for this integration ( $\mu\text{m}$ )
36	$\dot{R}$ Initial bubble wall velocity for this integration ( $\mu\text{m}/\mu\text{sec}=\text{m/sec}$ )

\*Numbers in parentheses are default values.



TABLE AI (Cont'd)

<u>XD Index</u>	<u>Storage Usage</u>
37	Fractional part of period which step size will not exceed (0.01)
38	Factor by which time step increased and reduced in minor adjustments (2)
39	$S_k$ Test factor on Runge-Kutta k's (0.03)
40	$f_o$ Bubble resonance frequency (kHz)
41	$C_o$ Sound speed, if zero, acoustic terms omitted from integration
42	SCT Scale factor for time array
43	SCR Scale factor for R array (this DR)
44	$\dot{S}CR$ Scale factor for $\dot{R}$ array (this DR)
45	$\ddot{S}CR$ Scale factor for $\ddot{R}$ array (this DR)
51	T(1) Beginning time in this DR ( $\mu$ sec)
52	R(1) Beginning radius in this DR ( $\mu$ m)
53	$\dot{R}$ (1) Beginning bubble wall velocity in this DR ( $\mu$ m/ $\mu$ sec)
54	T(Last) Last time in this DR ( $\mu$ sec)
55	R(Last) Last radius in this DR ( $\mu$ m)
56	$\dot{R}$ (Last) Last bubble wall velocity in this DR ( $\mu$ m/ $\mu$ sec)
57	$\Delta t_{lim}$ Maximum allowed $\Delta t$ ( $\mu$ sec)
58	$\Delta t_{min}$ Minimum $\Delta t$ , this DR ( $\mu$ sec)
59	$\Delta t_{max}$ Maximum $\Delta t$ , this DR ( $\mu$ sec)
60	Program name (Characters "GASBUBL5")

The DR can then be read into an integer array (IA) dimensioned (4,2048).  
The time in microseconds is given by

$$T(I) = IA(1,I) * SCLT \quad ,$$

the radius in micrometers by

$$R(I) = IA(2,I) * SCLR * RZERO \quad ,$$

$\dot{R}$  in micrometers per microseconds by,

$$RD(I) = IA(3,I) * SCLRD * RZERO \quad ,$$

and  $\ddot{R}$  by

$$RDU(I) = IA(4,I) * SCLRDD * RZERO \quad .$$

#### B. Resampled Output

The program GASRSMPL accepts the output of GASBUBL5 and produces an output tape of R,  $\dot{R}$ , and  $\ddot{R}$  array multiplexed as in GASBUBL5. The data are equally spaced in time as directed by control cards. The storage format is the same as that used by GASBUBL5 except that

- (1) only three arrays are multiplexed (time is absent),
- (2) ID(100) is 3,
- (3) XD(60) is "GASRSMPL," and
- (4) XD(42) to XD(59) have been changed to reflect the nature of the data stored on the following DR (e.g., XD(57), XD(58), and XD(59) are all set to the time step).

## REFERENCES

1. H. G. Flynn, "Physics of Acoustic Cavitation in Liquids," Physical Acoustics, W. Mason (ed.) (Academic Press, New York, 1964), Vol IB, p. 58.
2. M. S. Plesset and A. Prosperetti, "Bubble Dynamics and Cavitation" in Ann. Rev. Fluid Mech. 9, 145 (1977). This is a good recent review citing many references.
3. P. J. Westervelt, "Parametric Acoustic Array," J. Acoust. Soc. Am. 35, 535 (1963).
4. T. Muir and J. Blue, "Experiments in the Acoustic Modulation of Large-Amplitude Waves," J. Acoust. Soc. Am. 46, 227 (1969).
5. D. J. Dunn, M. Kuljis, and V. G. Welsby, "Nonlinear Effects in a Focused Underwater Standing Acoustic Wave System," J. Sound Vib. 2, 471 (1965).
6. V. G. Welsby and M. H. Safar, "Acoustic Nonlinearity Due to Microbubbles in Water," Acustica 22, 177 (1969).
7. J. S. M. Rusby, "The Onset of Sound Wave Distortion and Cavitation in Water and Sea Water," J. Sound Vib. 13, 257 (1970).
8. W. L. Konrad and M. B. Moffett, "Difference Frequency Radiation from a Cavitating Parametric Source," J. Acoust. Soc. Am. 55, 42E(A). See also Naval Underwater Systems Center Technical Memorandum No. TDIX-45-73, 1973.
9. T. G. Muir, "Nonlinear Parametric Transduction in Underwater Acoustics," 1974 Ultrasonics Symposium Proceedings, IEEE Cat. No. 74, CHO 896-ISU.
10. J. Clynch and R. Rolleigh, "Measurement of Enhanced Nonlinear Radiation in the Presence of Microbubbles," J. Acoust. Soc. Am. 55S 51(A) (1974). Also, Applied Research Laboratories Technical Memorandum No. 74-17 (ARL-TM-74-17), Applied Research Laboratories, The University of Texas at Austin, May 1974.
11. J. Lockwood and D. Smith, "Investigation of the Increase in Parametric Efficiency Due to Bubbles," J. Acoust. Soc. Am. 57S, 73(A) (1975). Also AMETEK Technical Report, 11-1354E-74-1, 20 August 1974.
12. J. R. Clynch and C. W. Dittman, "Bubble Enhanced Nonlinear Sound Generations," J. Acoust. Soc. Am. 59S, 88(A), 1976.



13. M. B. Moffett and W. L. Konrad, "Experiments with Cavitating Parametric Sources," J. Acoust. Soc. Am. 60S, 99(A), 1976.  
See also Naval Underwater Systems Technical Memorandum TM-316-568-76, 1 December 1976.
14. H. G. Flynn, "Cavitation Dynamics, I. A Mathematical Formulation," J. Acoust. Soc. Am. 57, 1379 (1975).
15. H. G. Flynn, "Cavitation Dynamics II. Free Pulsations and Models for Cavitation Bubbles," J. Acoust. Soc. Am. 58, 1160 (1975).
16. W. Lauterborn, "Numerical Investigation of Nonlinear Oscillations of Gas Bubbles in Liquids," J. Acoust. Soc. Am. 59, 282 (1976).
17. D. Epstein, "Effect of a Dense Gas Equation on the Expansion and Contraction of an Underwater Gas Bubble," J. Acoust. Soc. Am. 57, 1427 (1975).
18. J. K. Vaughan, M. W. Ohlendorf, and R. L. Sweet, "An Input-Output Software System for Signal Processing," Applied Research Laboratories Technical Report No. 74-6 (ARL-TR-74-6), Applied Research Laboratories, The University of Texas at Austin, 20 May 1974.

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